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M. A. Dorf, R. H. Cohen, A. N. Simakov, et al.

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On the applicability of the standard approaches for evaluating a neoclassical radial electric field in a tokamak edge region

M. A. Dorf,[1] R. H. Cohen,[1] A. N. Simakov,[2] and I. Joseph[1]
[1]*Lawrence Livermore National Laboratory, Livermore, California 94550, USA*
[2]*Los Alamos National Laboratory, Los Alamos, New Mexico 87544, USA*

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The use of the standard approaches for evaluating a neoclassical radial electric field E_r, i.e., the Ampere (or gyro-Poisson) equation, requires accurate calculation of the difference between the gyroaveraged electron and ion particle fluxes (or densities). In the core of a tokamak, the nontrivial difference appears only in high-order corrections to a local Maxwellian distribution due to the intrinsic ambipolarity of particle transport. The evaluation of such high-order corrections may be inconsistent with the accuracy of the standard long wavelength gyrokinetic equation (GKE), thus imposing limitations on the applicability of the standard approaches. However, in the edge of a tokamak, charge-exchange collisions with neutrals and prompt ion orbit losses can drive non-intrinsically ambipolar particle fluxes for which a nontrivial (E_r-dependent) difference between the electron and ion fluxes appears already in a low order and can be accurately predicted by the long wavelength GKE. The parameter regimes, where the radial electric field dynamics in the tokamak edge region is dominated by the non-intrinsically ambipolar processes, thus allowing for the use of the standard approaches, are discussed. [http://dx.doi.org/10.1063/1.4818777]

I. INTRODUCTION

It has been recently been demonstrated that the use of the standard approaches (i.e., the gyro-Poisson or flux-surface-averaged Ampere equation) that are utilized in the majority of drift-kinetic and gyro-kinetic codes for evaluating a long-wavelength axisymmetric neoclassical radial electric field E_r has limited validity in the core of a tokamak.[1,2] Briefly, the limitation comes from the fact that a difference between the ion and the electron flux-surface-averaged particle fluxes (that determines the radial electric field) appears only in high-order corrections to a local Maxwellian distribution (zero-order solution), whereas the standard drift-kinetic equation[3] or the long wavelength gyro-kinetic equation[4] may have insufficient accuracy to predict such high-order corrections. The low-order identity between the electron and ion fluxes for an arbitrary value of the radial electric field is a manifestation of the so-called *intrinsic ambipolarity*.[5] In the tokamak edge, however, charge-exchange collisions with neutrals[6,7] and ion orbit losses[8] can drive non-intrinsically ambipolar particle fluxes for which a nontrivial (E_r-dependent) difference between the electron and ion fluxes appears already in the low-order corrections (accurately predicted by the long wavelength GKE). Therefore, for the parameter regimes where the non-intrinsically ambipolar processes play a dominant role in determining the E_r-dynamics, or, equivalently, the non-intrinsically ambipolar (low-order) particle fluxes are dominant over the intrinsically ambipolar (high-order) particle fluxes, the standard approaches for evaluating E_r can be used. While both the charge-exchange collisions with neutrals and ion orbit losses are significant near the last closed flux surface (separatrix), their influence rapidly decreases toward the core region. Indeed, the neutral density exhibits an exponential decay due to ionization, and the ion orbit loss decreases due to the shift of the loss hole toward the high-energy tail of the particle distribution. On the other hand, the higher-order ambipolar particle losses are generally small, and it is therefore of particular interest to estimate the radial width of an edge layer where the non-intrinsically ambipolar losses are still dominant.

The paper is organized as follows. First, following the discussion in Ref. 2, we assess the higher-order "intrinsically ambipolar" particle fluxes and review the limitations of the standard approaches for evaluating a neoclassical radial electric field in a tokamak core. We then estimate the non-intrinsically ambipolar fluxes driven by the charge-exchange collisions with neutrals and the orbit ion losses, and evaluate the width of the layer in the edge where the standard approaches can be used.

II. LIMITATIONS OF THE STANDARD APPROACHES IN A TOKAMAK CORE

In order to understand the limitations of the standard approaches for evaluating a neoclassical radial electric field in a tokamak core, it is first important to discuss the accuracy of the standard "first-order" gyrokinetic equation[4]

$$\frac{\partial f}{\partial t} + \dot{\mathbf{R}}\frac{\partial f}{\partial \mathbf{R}} + \dot{v}_{||}\frac{\partial f}{\partial v_{||}} = \bar{C}[f]. \quad (1)$$

Here, Eq. (1) for the ion gyrocenter distribution function $f(\mathbf{R}, v_{||}, \mu)$ is written in parallel velocity ($v_{||}$)—magnetic moment (μ) coordinates, $\dot{\mathbf{R}} = v_{||}\mathbf{b} + \mathbf{v}_d$, $\dot{v}_{||} = -(1/m_i v_{||})\dot{\mathbf{R}}\cdot(Ze\nabla_{\mathbf{R}}\Phi + \mu\nabla_{\mathbf{R}}B)$, $\mathbf{v}_d$ is the drift velocity composed of the E × B drift and the magnetic drifts (i.e., curvature drift and ∇B drift), $\mathbf{R}$ is the gyrocenter position coordinate, $\bar{C}[f]$ denotes the gyro-averaged collision operator, $\mathbf{B} = B\cdot\mathbf{b}$ is the magnetic field with $\mathbf{b}$ denoting the unit vector along the field, e is the electron charge, Z and m_i correspond to the ion charge state and mass, respectively, and in what follows we assume $Z=1$ for simplicity. For the case of the long-wavelength neoclassical electrostatic potential variations with $e\varphi \sim T$ and $k_\perp \rho_i \sim \rho_i/L_p \ll 1$, the gyrokinetic electrostatic potential in Eq. (1) is given by $\Phi = \bar{\varphi} + (e\rho_i^2/2T_i)(\nabla_\perp \bar{\varphi})^2$, where ρ_i is the ion gyroradius, the bar over a variable denotes the gyro-average, $k_\perp^{-1}$ and L_p describe the length scale for variations of neoclassical electrostatic potential and plasma pressure, respectively; and finally, ion and electron temperatures of the same order are assumed, i.e., $T_i \sim T_e \sim T$.

The accuracy of Eq. (1) can be characterized by three dimensionless parameters: $\delta_B \sim \rho_i/L_B \ll 1$ corresponding to the accuracy of the magnetic drift velocity, $\delta_p \sim q\rho_i/L_p \ll 1$ describing the effects of finite drift-orbit width, and $\delta_E \sim (e\varphi/T)(k_\perp \rho_i) \ll 1$ describing the finite Larmor radius (FLR) polarization effects. Here, L_B is the characteristic length scale for variation of the magnetic field, typically being the order of the tokamak major radius, i.e., $L_B \sim R$, and q is the magnetic safety factor. Assuming the pressure length scale to be the order of the tokamak minor radius, $L_p \sim a$ (characteristic of a tokamak core region), the small parameters are related as $\delta_p/\delta_E \sim q$ and $\delta_B/\delta_p \sim B_\theta/B_\varphi$, where B_θ and B_φ denote the poloidal and toroidal components of the magnetic field, respectively. For the case of a large aspect ratio tokamak with $\varepsilon = a/R \ll 1$ and $q \sim 1$, it follows that $\delta_p/\delta_E \sim 1$ and $\delta_B/\delta_p \ll 1$. In the opposite limit of a spherical torus with $a \sim R$, one obtains $\delta_B \sim \delta_p \sim \delta_E$. While the standard gyro-kinetic equation [Eq. (1)] describes the finite-orbit-width effect to any order in δ_p, it takes into account only the first-order corrections to the magnetic drifts and therefore is accurate only through the first order in δ_B. Furthermore, the majority of present numerical codes do not provide accurate implementation of the gyro-averaged collision operator through order δ_E^3, and therefore here we assume that Eq. (1) is only accurate through the second order in δ_E. [It is also interesting to discuss the accuracy of Eq. (1) in the collisionless limit. First, we note that Eq. (1) and the subsidiary relations that follow do not include fourth-order FLR corrections, and therefore Eq. (1) cannot predict f to fourth order in δ_E^4. Also, for the case of turbulent perturbations with $e\varphi/T \ll 1$ and $k_\perp \rho_i \sim 1$, Eq. (1) also fails to predict δ_E^3 corrections. It is however intuitively appealing to assume for the case of the neoclassical electric field with $e\varphi/T \sim 1$ and $k_\perp \rho_i \ll 1$ that the collisionless limit of Eq. (1) is accurate through order δ_E^3. The detailed analysis of this subject is, however, outside the scope of the present work.]

For the case where the characteristic "radial" (i.e., normal to the magnetic flux surfaces) length-scales for variations of the ion temperature, L_T, and density, L_n, are large compared to the ion orbital excursion, a solution to the gyrokinetic equation (1) is close to a local Maxwellian distribution, F_M. Assuming, $L_T \sim L_n \sim L_p$ and $\delta_E \sim \delta_p$, it is straightforward to show for moderate (plateau) or weakly (banana) collisional regimes that

$$f_n = O(\delta_p^n)F_M, \tag{2}$$

where f_n is the nth-order correction to the local Maxwellian (zero-order) solution of Eq. (1), i.e., $f = F_M + f_1 + f_2 + \cdots$. However, if the second-order effects in δ_B were properly retained, the corrections to the zeroth-order Maxwellian solution would include the following missing terms:

$$f_2^{miss} = O(\delta_p\delta_B)F_M, \tag{3a}$$

$$f_3^{miss} = \{O(\delta_p^2\delta_B) + O(\delta_p\delta_B^2) + O(\delta_E^3)\}F_M. \tag{3b}$$

Therefore, a solution to Eq. (1) that only includes the corrections given in Eq. (2) is accurate through second order (i.e., $f_2 \gg f_2^{miss}$), if $\delta_p \gg \delta_B$. For instance, in the core of a spherical torus with $\varepsilon = a/R \sim 1$, we obtain $\delta_p \sim \delta_B$, and therefore already the second-order correction cannot be accurately evaluated. On the other hand, at the top of the pedestal of the DIII-D tokamak[9] corresponding to $\varepsilon \sim 0.3$, $q \sim 4$, $B \sim 1.8$ T, $L_p \sim a \sim 0.5$ m, $T_i \sim 400$ eV, and $m_i = 2m_p$, we obtain $\delta_B \sim 10^{-3}$, $\delta_p \sim 10^{-2}$, implying that although the second-order correction, f_2, can be accurately evaluated, the omitted second-order correction, f_2^{miss} [Eq. (3a)] is still larger than the third-order correction f_3 [in Eq. (2)] predicted by Eq. (1). Here, m_p denotes the proton mass. Furthermore, the adopted assumption of $\delta_p \sim \delta_E$ in our analysis implies that $f_3^{miss} \sim f_3$.

The limited applicability of the standard approaches [e.g., gyro-Poisson or the flux-surface-averaged Ampere equations] for the evaluation of the neoclassical radial electric field now comes from the fact that a distribution function needs to be accurately known through third order. Indeed, let us consider the standard flux-surface-averaged Ampere equation for long-wavelength variations of the electrostatic potential Φ that is often used in numerical simulations for evaluating a radial electric field[10]

$$\left[\langle|\nabla\psi|^2\rangle + 4\pi n_i m_i c^2\left\langle\frac{|\nabla\psi|^2}{B^2}\right\rangle\right]\frac{\partial^2\Phi}{\partial t\partial\psi} = 4\pi e\langle\Gamma_i\rangle. \tag{4}$$

Here, $n_i(\mathbf{R}) = (2\pi/m_i)\int B_\parallel^* d\mathrm{v}_\parallel d\mu f(\mathbf{R}, v_\parallel, \mu)$ is the ion gyro-density, ψ is the poloidal flux function, and the angular brackets $\langle\cdots\rangle$ denote the flux surface average. The second term on the left-hand-side (LHS) of Eq. (4) corresponds to the classical polarization current density, with $4\pi n_i m_i c^2/B^2 \gg 1$ for a typical tokamak plasma, and the right-hand-side (RHS) of Eq. (4) represents the neoclassical ion current

$$\langle\Gamma_i\rangle = \left\langle\int d^3\mathrm{v}(\mathbf{v}_d\cdot\nabla\psi)f\right\rangle = \frac{2\pi}{m_i}\left\langle\int B_\parallel^* d\mathrm{v}_\parallel d\mu(\mathbf{v}_d\cdot\nabla\psi)f\right\rangle, \tag{5}$$

where $B_\parallel^* \equiv \mathbf{b}\cdot(\mathbf{B} + (m_i \mathrm{v}_\parallel/e)\nabla\times\mathbf{b})$ is the Jacobian of the transformation from particle phase-space coordinates to the gyrokinetic variables. Note that the small neoclassical radial electron particle flux $\langle\Gamma_e\rangle \sim \nu_{ei}\rho_e^2 RB_\theta\nabla n_i \sim n_i\nu_{ii}(m_e/m_i)^{1/2}\rho_i^2 RB_\theta\nabla n_i$ is cancelled by the radial ion flux driven by the ion-electron collisions,[5] and therefore is not included in Eq. (4). Here, ν_{ei} and ν_{ii} are the electron-ion and ion-ion collision frequencies, and $\nabla\psi \sim RB_\theta$ is used. For consistency, the collision operator in Eq. (1) should not include weak ion-electron collisions. The radial electron current driven by the electron-electron collisions is much smaller than the corresponding ion current, and is neglected in Eq. (4) as well. Finally, Eq. (4) provides no information about the small, $O(\delta_p)$, poloidal variations of the neoclassical electrostatic potential. These poloidal variations can be determined separately, for instance, by making use of the quasi-neutrality condition along the magnetic field lines, which only requires accurate evaluation of f_1.

It is straightforward to show for a steady-state case (or, during slow, transport-time-scale evolution) that the ion particle flux in Eq. (5) generated by the first-order correction, f_1 [in Eq. (2)], vanishes[11]

$$\langle\Gamma_1\rangle \equiv \left\langle \int d^3\mathrm{v}(\mathbf{v}_d\cdot\nabla\psi)f_1 \right\rangle = -\left\langle \frac{I}{\Omega_i}\int d^3\mathrm{v}\,\mathrm{v}_{||}\bar{C}[f_1] \right\rangle = 0, \tag{6}$$

due to the momentum-conserving property of ion-ion collisions. Here, $I = RB_\varphi$ and $\Omega_i = eB/m_ic$ is the ion cyclotron frequency. Also, for an up-down symmetric tokamak, the second-order correction, f_2 [in Eq. (2)], produces zero particle flux as well, i.e., $\langle\Gamma_2\rangle = \langle\int d^3\mathrm{v}(\mathbf{v}_d\cdot\nabla\psi)f_2\rangle = 0.$[2,12] Therefore, an accurate calculation of the third-order correction, f_3, is required to obtain a nontrivial particle flux $\langle\Gamma_3\rangle$ and describe the radial electric field evolution [Eq. (4)], which can however be inconsistent with the accuracy of Eq. (1).

It is now important to estimate corrections to the ion particle flux that would be provided by the missing terms f_2^{miss} and f_3^{miss} [in Eqs. (3a) and (3b)]. First, we note that the missing corrections in f_2, i.e., f_2^{miss}, [Eq. (3a)] would not generate a nontrivial particle flux, even if properly retained. It follows from the well-established fact demonstrated in both fluid[13] and kinetic[12] theories that for an up-down-symmetric tokamak, the relaxation of the toroidal angular momentum occurs on the transport time scale, $O(\delta^2\omega_t)$. Here, $\omega_t = V_T/qR$ is the transit frequency, V_T is the ion thermal velocity, and $\delta\sim\delta_p\sim\delta_B$ is assumed for simplicity along with $\omega_t\sim\nu_{ii}$, where ν_{ii} denotes the ion-ion collision frequency. From radial force balance, one readily obtains the same-order relaxation time for the radial electric field, which [by virtue of Eq. (4)] requires that $\langle\Gamma_i\rangle$ be fourth order, so the exact $\langle\Gamma_2\rangle$ (evaluated with $f_2 + f_2^{miss}$) must be zero. Therefore, only the omitted corrections in f_3, i.e., f_3^{miss}, would generate nontrivial contributions to the ion particle flux. Assuming $\delta_p \sim \delta_E \geq \delta_B$, it follows that $\langle\Gamma_3\rangle$ misses the contributions, which are comparable to the one retained and can be estimated as

$$\langle\Gamma_3\rangle \sim \frac{\nu_{ii}}{\omega_t}\delta_B\delta_P^3 n_i V_T R B_\theta. \tag{7}$$

Here, $\nabla\psi \sim RB_\theta$ and $\mathrm{v}_d \sim \delta_B V_T$ is used, and the coefficient ν_{ii}/ω_t comes from the subsidiary expansion in collision frequency assuming weak collisionality regimes $\nu_{ii} \ll \omega_t$ typical of a tokamak core. That is, the first order correction, $f_1 = O(\delta_p)F_M$, should be considered as $f_1 = O[\delta_p(1+\nu_{ii}/\omega_t + \nu_{ii}^2/\omega_t^2 + \cdots)]F_M$, where the lowest-order (collisionless) contribution does not contribute to the particle flux.

We now discuss another standard approach for evaluating E_r in neoclassical simulations, which utilizes the long wavelength limit, $k_\perp\rho_i \ll 1$, of the gyro-Poisson equation[14]

$$\frac{1}{4\pi e}\Delta^2\Phi + \frac{e}{m_i\Omega_i^2}\nabla_\perp\cdot(n_i\nabla_\perp\Phi) = n_e^{ad} - n_i, \tag{8}$$

with a Boltzmann (in the linear limit, adiabatic) model for electrons

$$n_e = \langle n_{i0}\rangle \frac{\exp[e\Phi/T_e(\psi)]}{\langle\exp[e\Phi/T_e(\psi)]\rangle}, \tag{9}$$

where $n_i = n_i(\mathbf{R}) = (2\pi/m_i)\int B_{||}^* d\mathrm{v}_{||}d\mu f(\mathbf{R},\mathrm{v}_{||},\mu)$ is the ion gyro-density and $n_{i0} = n_i(t=0)$. Note that although the ion gyro-density, n_i, is Eqs. (8) and (9) should formally be evaluated at the spatial coordinate $\mathbf{x}$, and not the gyro-center coordinate $\mathbf{R}$, a small difference of $n_i(\mathbf{x}) - n_i[\mathbf{R}(\mathbf{x})] = O(k_\perp^2\rho_i^2)n_i \ll n_i$ can be neglected. Indeed, retaining these corrections in Eqs. (8) and (9) yields $\langle n_e^{ad} - n_i\rangle = \langle n_i - n_{i0}\rangle + O(k_\perp^2\rho_i^2)\langle n_i - n_{i0}\rangle$, where the second term $O(k_\perp^2\rho_i^2)\langle n_i - n_{i0}\rangle$, related to the discrepancy between $n_i(\mathbf{x})$ and $n_i(\mathbf{R})$, is much smaller than the left-hand-side of Eq. (8). Applying flux surface averaging to Eq. (8), taking a time derivative and making use of the gyro-kinetic density moment equation (continuity equation), i.e.,

$$\frac{\partial\langle n_i\rangle}{\partial \mathrm{t}} = -\frac{\partial}{\partial\psi}\langle\Gamma_i\rangle = -\frac{\partial}{\partial\psi}\left\langle \int \mathrm{B}_{||}^* d\mathrm{v}_{||}d\mu(\mathbf{v}_d\cdot\nabla\psi)f \right\rangle, \tag{10}$$

we obtain Ampere's law [Eq. (4)]. Therefore, the limitations on the applicability of the gyro-Poisson equation [Eq. (8)] for evaluating E_r in a tokamak core are the same as for Ampere's law [Eq. (4)]. Physically, the approach utilizing gyro-Poisson's equation relies on the accurate evaluation of $\langle n_i\rangle$ evolution, which, in turn, is directly related to the accurate evaluation of $\langle\Gamma_i\rangle$. It is straightforward to show that Ampere's law [Eq. (4)] also follows from the gyro-Poisson equation for the case of kinetic electrons since the small radial electron and ion currents associated with the electron-ion collisions cancel each other,[5] thus imposing the same applicability limitations for that case.

It is worth noting that methods for evaluating a radial electric field that require accurate evaluation of a distribution function only through order n = 2,[2,15] and even n = 1 (Ref. 12) are being developed as well. However, they are valid in limited parameter regimes, and are challenging for numerical implementation.

III. APPLICABILITY OF THE STANDARD APPROACHES IN A TOKAMAK EDGE

The edge of a tokamak is distinguished by the presence of non-intrinsically ambipolar processes such as charge-exchange collisions with neutrals and prompt ion orbit losses. The ion particle flux driven by charge-exchange friction with neutrals can be estimated from Eq. (6) as follows:

$$\langle\Gamma_{chx}\rangle = -\left\langle \frac{I}{\Omega_i}\int d^3\mathrm{v}\,\mathrm{v}_{||}\bar{C}_{chx}[f,f_n] \right\rangle \sim \frac{m_ic}{e}R\langle\nu_{chx}\rangle n_i(V_{i\varphi} - V_{n\varphi}), \tag{11}$$

where $\bar{C}_{chx}[f,f_n]$ is the gyroaveraged operator for charge-exchange collisions, $\nu_{chx} = (\sigma\mathrm{v})_{chx}n_n$, n_n is the neutral density, $(\sigma\mathrm{v})_{chx}$ is the charge-exchange reactivity, and $V_{i\varphi}$ and $V_{n\varphi}$ are the toroidal ion and neutral flow velocities, respectively. It is evident that the Ampere or gyro-Poisson equations can be applied for evaluating the radial electric field in a tokamak edge region provided $\langle\Gamma_{chx}\rangle \gg \langle\Gamma_3\rangle$. This condition needs to be satisfied as the system relaxes to a steady state, where, in the absence of other particle sources or sinks, $\langle\Gamma_{chx}\rangle$ becomes equal to $\langle\Gamma_3\rangle$. In order to estimate the friction force between ions and neutrals during the relaxation, one needs to take into account the dynamics of neutrals. Considering the flux-surface averaged toroidal angular momentum equation for the neutral species and neglecting the inertial term, it follows for the axisymmetric case that

$$\langle \bar{R} m_n n_n \nu_{ni}(V_{i\varphi} - V_{n\varphi})\rangle = \langle \bar{R}\hat{\varphi}\cdot\nabla\cdot\Pi_n\rangle, \qquad (12)$$

where $\bar{R}$ is the major radius coordinate, Π_n is the neutral viscous tensor, $\nu_{ni} = \nu_{chx}(n_i m_i)/(n_n m_n)$ is the neutral-ion charge-exchange collision frequency, and m_n is the neutral mass. For simplicity, here, we estimate the right-hand-side of Eq. (12) as $\langle \bar{R}\hat{\varphi}\cdot\nabla\cdot\Pi_n\rangle \sim R\mu_n V_{n\varphi}/l_n^2$, where μ_n is the neutral viscosity, $T_n \sim T_i$ is the neutral temperature, and l_n is the characteristic perpendicular length scale for variations of the neutral toroidal flow velocity, which is the order of or larger than that of the ion toroidal flow velocity, l_i. For a strongly collisional case with $\lambda_n \ll l_n$, we adopt $\mu_n \sim n_n T_n/\nu_{ni}$ and readily obtain from Eq. (12) that $(V_{i\varphi} - V_{n\varphi})/V_{n\varphi} \sim \lambda_n^2/l_n^2 \ll 1$, where $\lambda_n \sim V_{Tn}/\nu_{ni}$ is the neutral mean-free-path, and $V_{Tn} = \sqrt{(2T_n/m_n)}$ is the thermal neutral velocity. It then follows that the ion and neutral velocities are approximately equal, $V_{n\varphi} \approx V_{i\varphi}$, $l_n \approx l_i$, and the particle flux in Eq. (11) is given by $\langle\Gamma_{chx}\rangle \sim (m_i c/e)\langle\nu_{chx}\rangle(\lambda_n^2/l_n^2)n_i V_{i\varphi}$. More detailed analysis of the strongly collisional case can be found in Ref. 16. In the opposite, weakly collisional limit, $\lambda_n \geq l_i$, the neutrals cannot respond to rapid variations in the ion flow velocity. In this limit, $\mu_n \sim m_n n_n l_n V_{Tn}$, and we obtain from Eq. (12) that $V_{i\varphi} - V_{n\varphi} \sim V_{i\varphi}$, assuming the ion flow velocity variations across the tokamak edge, $\Delta V_{i\varphi}$, are the order of $V_{i\varphi}$ consistent with the experimental observations in Ref. 9. The particle flux in a weakly collisional regime can therefore be estimated as $\langle\Gamma_{chx}\rangle \sim (m_i c/e)\langle\nu_{chx}\rangle n_i V_{i\varphi}$. Adopting near-separatrix parameters characteristic of the DIII-D tokamak in the H-mode confinement regime,[9] $T_i \sim 200\,\mathrm{eV}$, $n_i \sim 10^{19}\,\mathrm{m}^{-3}$, $m_n \sim m_i$, and assuming $(\sigma v)_{chx} \sim 3\times10^{14}\,\mathrm{m^3s^{-1}}$,[6] it follows that $\lambda_n \sim 50\,\mathrm{cm}$, which is the order of the distance between the divertor plates and the X point. The length scale for variations of the diamagnetic-size toroidal ion flow velocity is the order of several centimeters,[9] and therefore this parameter regime corresponds to a weakly collisional case, with $V_{i\varphi} - V_{n\varphi} \sim V_{i\varphi}$. It now follows that the Ampere or gyro-Poisson equations can be applied for evaluating a radial electric field in a tokamak edge region provided

$$\frac{B_\varphi}{B_\theta}\frac{(\sigma v)_{chx}\langle n_n\rangle}{\Omega_i} \gg \frac{\nu_{ii}}{\omega_t}\delta_B\delta_p^2, \qquad (13)$$

where a diamagnetic level of the ion toroidal flow velocity, $V_{i\varphi} \sim \delta_p V_T$, has been adopted. For the parameters of the DIII-D tokamak pedestal region,[9] the inequality in Eq. (13) can be expressed as $n_n \gg 3\times10^{12}\,\mathrm{m}^{-3}$, where $\nu_{ii}/\omega_t \sim 0.1$ was assumed. Note that while a typical neutral density can be as high as $\sim10^{19}\,\mathrm{m}^{-3}$ at the divertor plates, it rapidly decreases toward the core region due to ionization.[6,7] Numerical studies show, however, that the condition in Eq. (13) can be well-satisfied even at the top of the pedestal.[7]

We now discuss the effects of the non-intrinsically ambipolar ion orbit losses[8,9,17] shown in Fig. 1. Here, let us consider the electron and ion species being distributed initially according to local Maxwellian distributions with equal charge densities. Furthermore, we assume a weakly collisional regime with $\nu_{ii} \ll \varepsilon^{1/2}\omega_t$. As the system relaxation occurs, the ion orbit losses produce a large non-intrinsically ambipolar ion particle flux (the corresponding electron particle flux is much smaller), thus leading to the rapid generation of a radial electric field in accordance with Eq. (4) [or, Eq. (8)]. This electric field will, in turn, suppress the ion orbit losses by shifting them toward the high-energy tail of the ion distribution due to development of a potential barrier and E_r-shear. It is important to note that if the $\langle\Gamma_{loss}\rangle$ dependence on the radial electric field is represented by a monotonically decreasing function, e.g., $\langle\Gamma_{loss}\rangle = \Gamma_{crit}\exp(-E_r/E_{crit})$, then, in the absence of other non-intrinsically ambipolar processes, the final steady-state value of the radial electric field can only be determined with limited (e.g., logarithmic) accuracy $E_r = E_{crit}\ln(\Gamma_{crit}/\langle\Gamma_3\rangle)$. However, if other non-intrinsically ambipolar processes are present, e.g., charge-exchange collision with neutrals, then the electric field can be accurately determined from the condition $\langle\Gamma_{loss}\rangle + \langle\Gamma_{chx}\rangle = 0$, provided $\langle\Gamma_{loss}\rangle, \langle\Gamma_{chx}\rangle \gg \langle\Gamma_3\rangle$.

In order to evaluate the threshold in the condition $\langle\Gamma_{loss}\rangle \gg \langle\Gamma_3\rangle$, it is sufficient to estimate small suprathermal ion orbit losses from the flux surfaces corresponding to

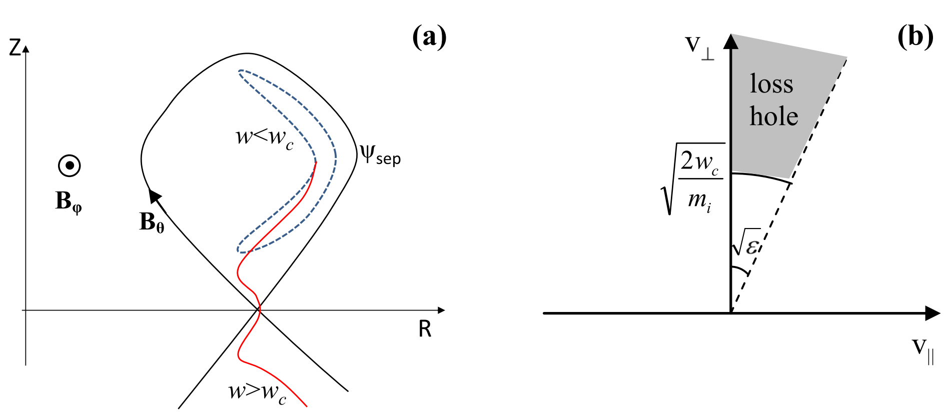

FIG. 1. Prompt ion orbit losses (simplified model). (a) Shown schematically are the lost (solid red curve) and confined (dashed blue curve) ion trajectories corresponding to $w > w_c$ and $w < w_c$, respectively. The ∇B drift is assumed to be directed downward. (b) Schematic of a loss hole in the velocity phase-space corresponding to the outer midplane.

$w_c \gg T_i$ (Fig. 1). Here, $w_c(\psi)$ denotes a critical energy at which the particle's banana orbit width, $\Lambda_c(w_c)$ becomes comparable to the distance between the flux surface under consideration, ψ, and the separatrix, i.e.,

$$\Lambda_c(w_c) \sim \sqrt{\varepsilon}\rho_{i\theta}\sqrt{\frac{w_c}{T_i}} \sim \frac{\psi - \psi_{sep}}{B_\theta R}, \tag{14}$$

where $\rho_{i\theta} = V_T m_i c/(eB_\theta)$ is the ion thermal poloidal gyroradius. Note that in deriving Eq. (14), we neglected the effects of quasi-stationary E_r on the suprathermal ion trajectories. Indeed, even for the case of the H-mode confinement regime, the potential variations $\Delta\varphi \sim T_i/e$ that occur in the pedestal region with a characteristic length-scale of order $\rho_{i\theta}$ should not significantly affect the suprathermal ion dynamics. Also, for simplicity of the present calculations, we assume that lost ion trajectories correspond to $w > w_c$ (Fig. 1). During the initial relaxation period [of the order of transient time scale, $\tau_{tr} \sim \varepsilon^{-1/2} qR/\sqrt{w_c/m_i}$], the collisionless orbit losses are primarily attributed to depletion of the initial particle distribution in the high-energy loss-hole regions of the ion phase-space. After this short initial stage, the losses are due to the slow collisional ion scattering into the loss holes. Note that the loss holes remain nearly empty provided $\nu_{ii}(w_c) \ll \varepsilon^{1/2}\sqrt{(w_c/m_i)}/(qR)$. Similarly to Eqs. (6) and (11), the particle flux associated with the ion orbit loss can be related during the quasi-stationary evolution of the system to the corresponding sink of the flux-surface averaged toroidal angular momentum attributed to the collisional population of the suprathermal loss holes

$$\langle \Gamma_{loss}\rangle \sim (I/\Omega_i) \mathrm{v}_{||}^{\mathrm{loss}} \dot{n}_{loss}. \tag{15}$$

Here, $\mathrm{v}_{||}^{\mathrm{loss}} \sim \sqrt{\varepsilon}(w_c/m_i)^{1/2}$ is the parallel velocity of the lost ions and $\dot{n}_{loss} \sim (T_i/w_c)^{3/2}\nu_{ii} n \exp(-w_c/T_i)$ is the particle loss rate due to the scattering into the loss hole, where ν_{ii} denotes the collision frequency corresponding to a thermal ion and the factor of $(T_i/w_c)^{3/2}$ comes from the energy dependence of the ion-ion collision frequency. Note that the losses from the outer midplane are used in Eq. (15) to represent the corresponding flux-surface averaged value. It is straightforward to show that ion orbit loss from that location is dominant for a given magnetic flux surface.[9] It now readily follows that:

$$\langle \Gamma \rangle_{loss} \sim n\nu_{ii}\Lambda_0 \left(\frac{\Lambda_0}{\Lambda_c}\right)^2 \exp\left(-\frac{\Lambda_c^2}{\Lambda_0^2}\right) RB_\theta, \tag{16}$$

where we introduced the thermal banana orbit width $\Lambda_0 = \sqrt{\varepsilon}\rho_{i\theta}$. The non-intrinsically ambipolar ion particle flux associated with the ion orbit losses [Eq. (16)] is dominant over the high-order flux in Eq. (7) within the region determined from

$$\left(\frac{\Lambda_0}{\Lambda_c}\right)^2 \exp\left(-\frac{\Lambda_c^2}{\Lambda_0^2}\right) \gg \sqrt{\varepsilon}\delta_p^3. \tag{17}$$

For the parameters characteristic of the DIII-D tokamak pedestal region,[9] the threshold in the inequality in Eq. (16) corresponds to $\Lambda_c = 3.5\Lambda_0 = 4.2$ cm.

It is also interesting to discuss the initial collisionless ion losses that occur during the short transient time, $\tau_{tr} \sim \varepsilon^{-1/2} qR/\sqrt{w_c/m_i}$, and become increasingly important in the region of close proximity to separatrix. Indeed, as the size of the loss-holes increases toward the separatrix, these prompt losses can become sufficiently large to generate a strong electrostatic potential barrier, $\Delta\varphi \geq T_i$, so as to substantially suppress the losses. After the initial transient stage, the collisional losses will continue to determine the E_r-dynamics. In order to show that the initial transient losses can indeed produce significant electrostatic potential variations, we estimate the largest possible radial electric field, E_{tr}^{loss}, generated due to the collisionless ion losses, assuming that the loss holes are completely emptied out during the initial transient time period. Making use of Eq. (4), we obtain

$$(m_i n c^2/B^2) E_{tr}^{loss}(\Psi) \sim e\Delta N_{tr}^{loss}(\Psi)/A(\Psi). \tag{18}$$

Here, $\Delta N_{tr}^{loss}(\Psi) \sim \int_0^\Psi |\nabla\psi|^{-1} A d\psi \sqrt{\varepsilon} n \exp(-w_c/T_i) \sim \sqrt{\varepsilon} n A(\Psi)(\Lambda_0^2/\Lambda_c)\exp(-\Lambda_c^2/\Lambda_0^2)$ is the initial number of ions inside the loss-holes integrated over a plasma volume bounded by the magnetic flux $\Psi = 0$ corresponding to the magnetic axis, $A(\Psi)$ is the area of the magnetic flux surface, and we made use of $\int_x^\infty \exp(-t^2)dt \sim x^{-1}e^{-x^2}$ for $x > 1$. From Eq. (18), it now follows that the variation of the electrostatic potential within a single thermal-banana-width, $\Lambda_c \sim \Lambda_0$, layer (adjacent to the separatrix) is given by $e\Delta\varphi/T_i \sim q^2/\sqrt{\varepsilon} > 1$. Therefore, the potential barrier can become strong enough to significantly suppress the ion losses and prevent a pronounced depletion of the initial ion distribution inside the loss-hole regions near the separatrix.

Finally, we note that the ion orbit-loss process can, in principle, introduce large nonlinear perturbations to the edge of a tokamak. Therefore, the properties of the edge steady state can depend on the assumptions of the initial plasma distribution inside the loss-hole regions of the phase space. As an illustrative example, in the present work, we adopt a local Maxwellian distribution to describe the initial state of the system, because it is often used in numerical simulations to initialize a tokamak edge.

IV. DISCUSSION

The standard approaches [i.e., Eq. (4) or (8)] for evaluating a long-wavelength radial electric field can be applicable in a tokamak edge, where low-order non-intrinsically ambipolar particle fluxes associated with the charge-exchange collisions with neutrals and prompt ion orbit losses are dominant over intrinsically ambipolar high-order fluxes (also present in a tokamak core). The non-intrinsically ambipolar fluxes are assessed, and the width of a layer inside the last closed flux surface where the standard approaches can be used is estimated. However, the aforementioned standard approaches have limited validity within the framework of standard gyrokinetics to evaluate the long-wavelength radial electric field in the tokamak core.

We also note that the problem of E_r relaxation is closely related to the problem of plasma toroidal rotation. Indeed, the radial electric field and the ion toroidal velocity, V_φ, are directly related through radial force balance; and physically, the condition $\langle\Gamma_3\rangle \ll \langle\Gamma_{chx}\rangle, \langle\Gamma_{loss}\rangle$ means that the relaxation of both E_r and V_φ occurs on a time scale faster than the transport time scale. It also means that the relaxation of the toroidal angular momentum is dominated (over the neoclassical viscosity) by the torque provided by charge-exchange collisions [Eq. (11)] with neutrals or large orbit losses [Eq. (15)]. Accordingly, the applicability of the standard approaches can be numerically analyzed for a given simulation by diagnosing the time evolution of the flux-surface averaged gyrokinetic toroidal angular momentum, $P_\varphi = \langle \int B^*_{||} dv_{||} d\mu (I/B) v_{||} f\rangle$. The standard approaches can be used in the edge region of an up-down symmetric tokamak when the time evolution of P_φ (with a diamagnetic level of the toroidal flow velocity, $V_\varphi \sim \delta_p V_T$) occurs on a time scale shorter than $\tau_\varphi = (\dot{P}_\varphi/P)^{-1} \sim [(\Omega_i/\delta_p V_T)(\Gamma_3/nRB_\theta)(B_\varphi/B_\theta)]^{-1} \sim [\delta_p^2 \nu_{ii} q (B_\varphi/B_\theta)]^{-1}$, i.e., the transport time scale.

Finally, we would like to emphasize that the present studies are performed for the case of axisymmetric particle transport. In order to generalize the analysis to including turbulent transport one needs to perform detailed studies to determine the order of the distribution function correction for which the difference between turbulent electron and ion particle fluxes becomes nontrivial (for an arbitrary value of the radial electric field). If the order is sufficiently high and is inconsistent with the accuracy of the gyrokinetic equation, then the conditions in Eqs. (13) and (17) need to be modified to include the maximum of the high-order (non-intrinsically ambipolar) turbulent and neoclassical particle fluxes.

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