Gyrokinetic Simulations of Drift-Wave Instabilities in Flow-Stabilized Z-pinch plasmas

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45th ICOPS
Denver, Colorado; June 24-28, 2018

LLNL-POST-752551

This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under contract DE-AC52-07NA27344 and supported by the Laboratory Directed Research and Development Program (18-ERD-007)

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Outline

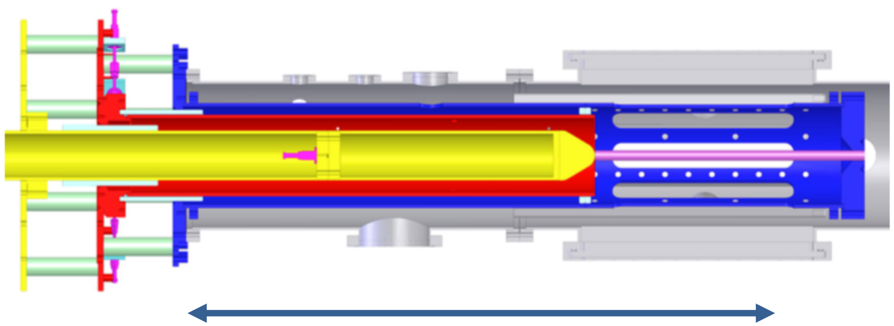
- Motivation
- Applying the COGENT code to simulations of electrostatic ion scale micro-turbulence in sheared-flow stabilized Z-pinch plasmas
- Verification studies
  - Linear analysis
  - Nonlinear analysis / Zonal flows
- First results from COGENT simulations of a FUZE-like Z-pinch system
- Conclusions

Motivation

- Recent discovery and demonstration of the shear-flow stabilization effect enables emerging of compact Z-pinch devices for fusion-energy applications

- The success of the concept scaling to higher energy-density regimes requires efficient predictive modeling capabilities

sheared-flow Z-pinch schematic

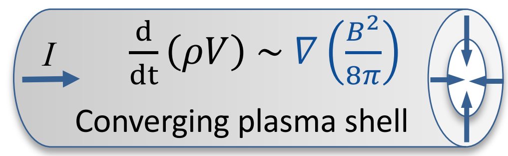


~ 1m

Z-pinch configurations for fusion energy applications

<u>Inertial (fast) Z-pinches</u>

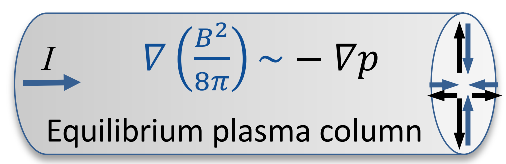
Fast magnetic compression



Imploding Liners, MagLIF, Dense Plasma Focus (DPF)

<u>Bennett-pinch equilibria</u>

Plasma pressure = Magnetic pressure



Earliest concept; revived interest due to recent flow-stabilization demonstration

Emerging Z-pinch systems have great promise; progress suffers due to lack of computational tools

Goal: need to model microturbulence in flow-stabilized Z-pinch plasmas

Why not use existing computational tools presently available within Z-pinch community?

- Fluid models lack fidelity
  - problems of interest are weakly collisional / kinetic effects are important

- Fully-kinetic description can be too computationally expensive
  - follows particle gyro-motion
  - requires resolving fast gyro-period for explicit schemes such as those often used for Z-pinch modeling efforts

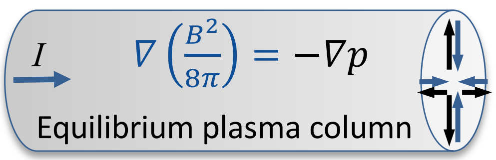
- Gyrokinetic (gyro-averaged) formulation reduces the computational demands while retaining important kinetic physics
  - developed and extensively used in the tokamak community
  - implemented in the HPC code COGENT (LLNL)

We exploit the gyrokinetc formalism to provide an efficient and accurate computational tool for modeling high energy density Z-pinch systems

Background: stability properties of Bennet Z-pinch equilibria

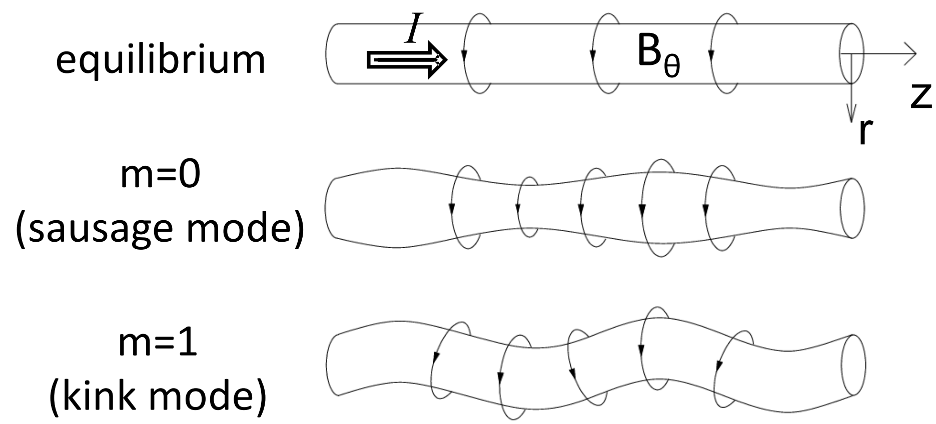
Bennet-pinch equilibria

Plasma pressure = Magnetic pressure



- Simple and compact geometry
- Efficient utilization of the magnetic field, $\beta = \frac{plasma\ pressure}{magnetic\ pressure} \sim 1$
- Ohmically heated DT plasma
- Attractive for fusion–energy applications

Bennet-pinch is MHD unstable



- Sausage mode can be stable in weak pressure-gradient systems
  - comes at the expense of increased size
  - pressure control is difficult
- Kink mode cannot be stabilized by controlling pressure

MHD stability issues motivated the fusion community's interest in more complex magnetic confinement concepts (e.g., low-β tokamak)

Sheared axial plasma flow can stabilize Z-pinch

Early ideas of Z-pinch stabilization

- External axial B-field – opens field lines, leads to enhanced heat loss
- Close-fitting conducting wall – must be very close $r_w/a < 1.2$, incompatible with hot plasma

More recent idea: Flow Stabilized Z-Pinch (FSZP)

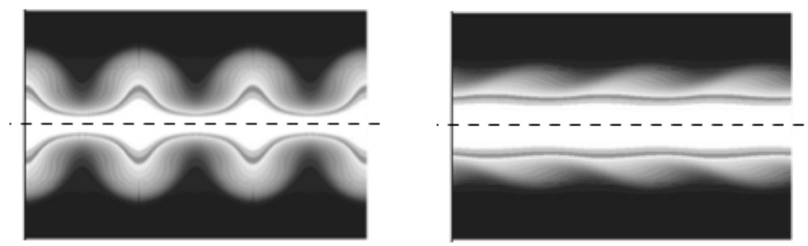
Theory: m=1 mode is stabilized by an axial flow with a <u>modest</u> shear

$$\frac{dV_z}{dr} \geq 0.1 k_z V_A$$

V_A - Alfven velocity
$k_z \sim a^{-1}$ axial wave vector

Shumlak et al, PRL 1995, 2001

Simulations: m=0 mode is stabilized by a sheared flow

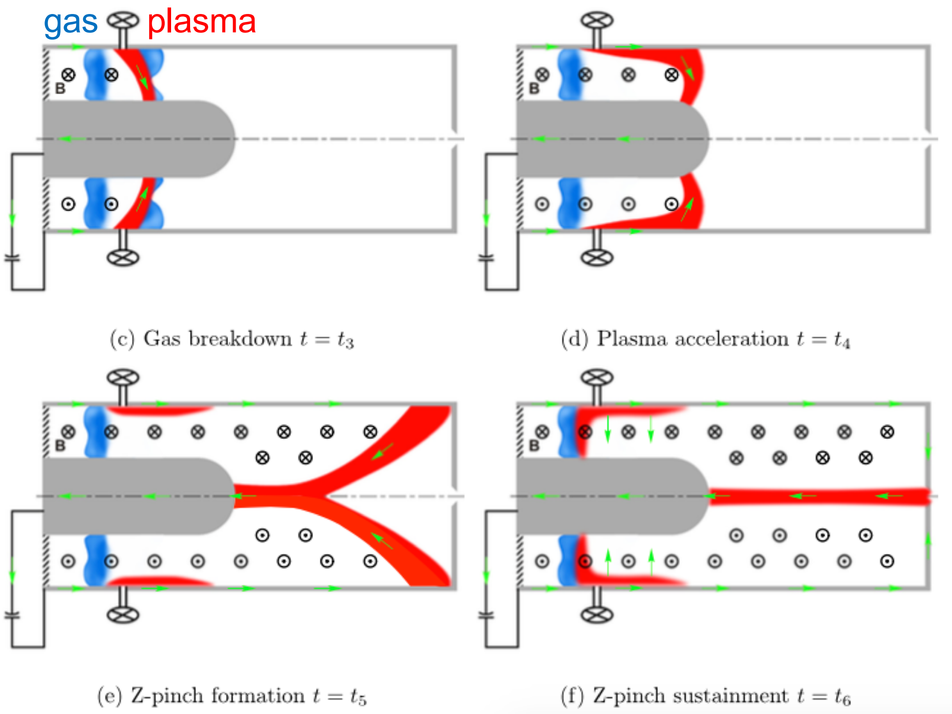


No shear $V_z = 0$ | Sheared flow $\Delta V_z = 0.2 V_A$

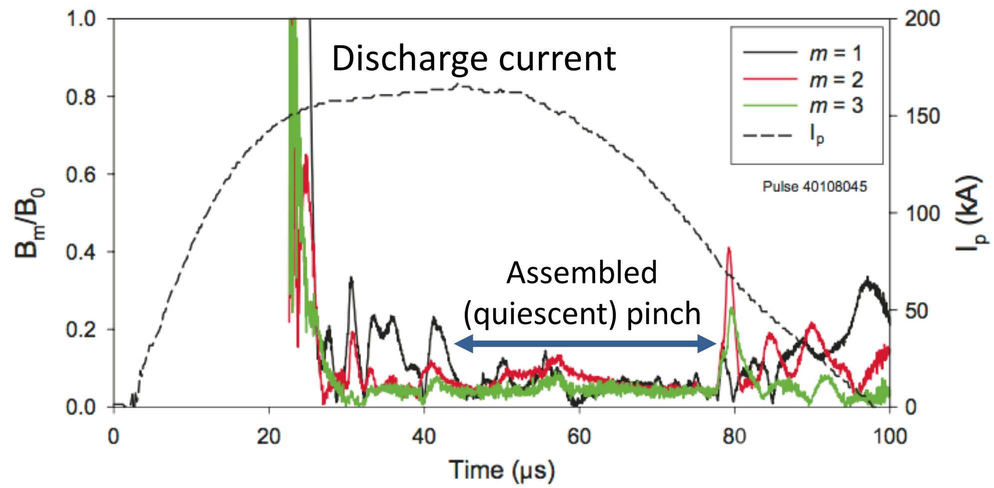
The stabilizing effect is a phase-mixing at different radii in the pinch

FSZP concept has been experimentally demonstrated on a 50kA facility at U. Washington

Schematic of Z-pinch formation



Magnetic probe data



50 kA (1 m long x 1 cm diameter) Z-pinch column is shown to be stable for 40 μs; MHD growth rate ~ 20 ns

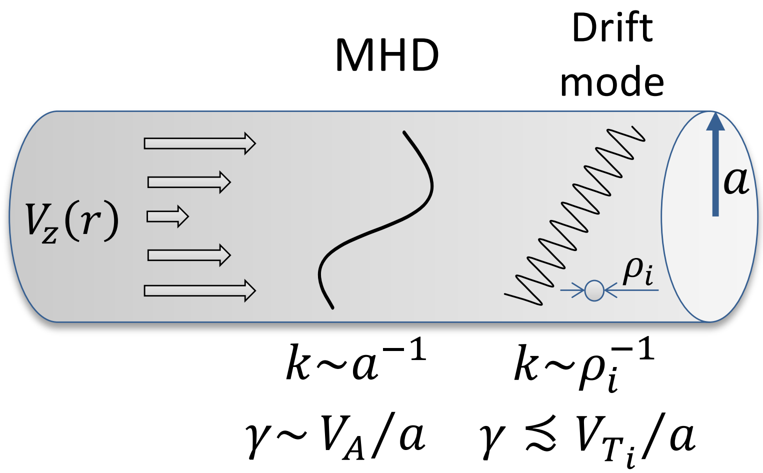
Shumlak et al, Nucl. Fusion **49**, 075039 (2009)

The FSZP concept is currently being extended to I~300 kA U~20 kV, τ~100's μs

- intermediate step toward MA-scale reactor
- produce useful intensities of neutrons and X-rays

FSZP application: while a sheared flow improves MHD stability, weaker microturbulence remains

- Short-scale micro-instabilities ($ka \gg 1$) cannot be stabilized by a large-scale ($\kappa_V a \sim 1$) velocity shear
- Microturbulence yields anomalous transport; of particular importance is anomalous viscosity that can act over time to reduce velocity shear and thus degrade global confinement
- Anomalous transport becomes increasingly important at higher currents and energy density
- Axisymmetric microturbulence is represented by the short-wavelength limit of the ideal m=0 MHD mode and the drift-type entropy mode



Anomalous radial transport properties are regulated by entropy mode turbulence

- While an ideal MHD sausage (m=0) mode can be stabilized by reducing pressure gradients (e.g., $L_p/R > 2/7$ for $\beta \ll 1$), a weaker drift-type ion-scale ($k_\perp \rho_i \lesssim 1$) axisymmetric (m=0) *entropy* mode remains
- The entropy mode is driven (unstable) by a combination of pressure-gradient and unfavorable magnetic curvature effects
- In the long-wavelength limit $k_\perp \rho_i \ll 1$ the total plasma pressure is unperturbed to a leading order, $\delta p = \delta p_e + \delta p_i$ =0, ($\delta n_e = -\delta n_i$, $\delta T_e = -\delta T_i$), hence the entropy is perturbed, $\delta s \neq 0$. [This is in contrast to ideal MHD, which assumes adiabatic equation of state and $s = const$]
- Owing to $\delta p = 0$, in the long-wavelength limit, the linear mode is of electrostatic character for arbitrary β

A kinetic description is required to describe entropy mode microturbulence for the parameters characteristic of FSZP plasmas

FSZP microturbulence is ideally suited for the gyrokinetic formalism

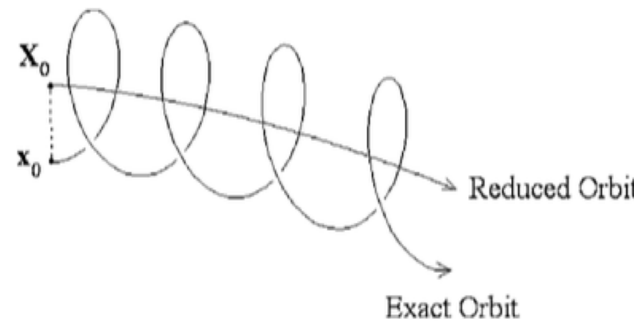
Why do we need kinetics?

- collisions are weak, $\gamma\tau_{collis} \gg 1$
- perpendicular scales are short, $k_\perp \rho_i \sim 1$

When can we use gyrokinetics?

- long time scales, $\gamma^{-1}\omega_{ci} \gg 1$
- large equilibrium scales for **E** and **B**, $\rho_i \ll L_{E,B}$
- small amplitude of perturbations, $e\tilde{\phi}/T \ll 1$ for $k_\perp \rho_i \sim 1$

Gyrokinetic approximation



- removes fast time scales
  10^4 speed-up VS explicit kinetics
- 6D→5D (discards gyrophase)
- has a small error $O\left(\rho_i^2/L_{E,B}^2\right)$

Previous gyrokinetic studies of entropy mode*

- Z-pinch geometry was used as simple testbed for tokamak tools and physics
- only low-beta / no axial flows / local flux-tube regimes were considered

*P. Ricci et al., PRL (2006); A. Navarro et al., arXiv:1512.06058 (2015).

Gyrokinetic formalism is implemented in LLNL's code COGENT

High-order (4th-order) finite-volume Eulerian gyrokinetic code

Physics team (DOE/FES)

LLNL, UCSD

Use the code to study magnetized plasma dynamics

Physics models

- Multispecies gyrokinetic equations
- Self-consistent electrostatic potential
- Collisions (including full Fokker-Plank)

Current limitations

- Electrostatic perturbations
- Long-wavelength limit ($k_\perp \rho_i < 1$)

M. Dorf et al., PoP (2016), Contrib. Plasm. Phys (2018)

Math teams (DOE/ASCR)

LLNL, LBNL

Consider the code as a testbed for their algorithmic advances

Math algorithms

- Complex coordinate systems
- Fast solvers / HPC
- Advanced time integrators (ImEx)

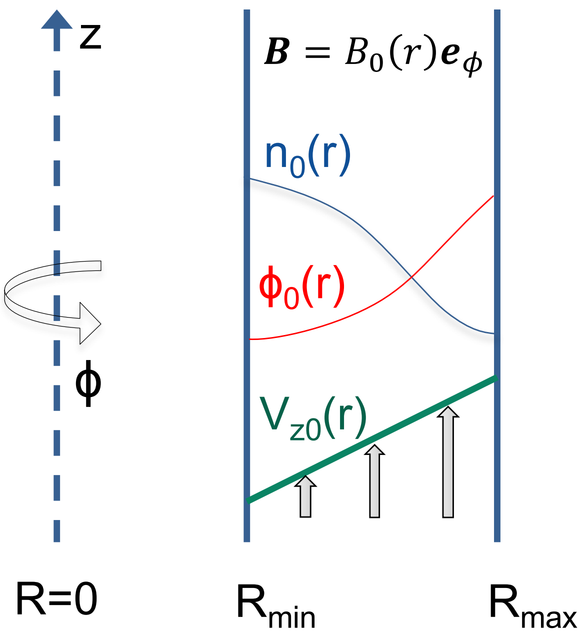
Moving toward inclusion of

- Finite-β EM perturbations
- FLR ($k_\perp \rho_i \sim 1$) corrections

M. Dorr et al., JCP (2018) in press

Present simulation model: azimuthally-symmetric 4D long-wavelength gyrokinetics

Simulation domain
(pinch periphery)



$$V_z = -\frac{c\nabla\phi}{B_0} - \frac{c\nabla p}{qB_0 n}$$

For given n_{i0} and V_{z0}, adjust n_{e0} to obtain required ϕ_0

Kinetic equation $\alpha = i, e$

$$\frac{\partial f_\alpha}{\partial t} + \dot{\boldsymbol{R}}_\alpha \nabla f_\alpha + \dot{v}_{\parallel,\alpha}\frac{\partial f_\alpha}{\partial v_\parallel} = 0$$

Gyro-Poisson (quasi-neutrality) equation

$$\nabla_\perp\left(\frac{nm_i c^2}{B^2}\nabla_\perp\phi\right) = -\sum_\alpha q_\alpha \int \left(\frac{2\pi B}{m_\alpha}\right) f_\alpha dv_\parallel d\mu$$

Particle guiding-center motion

$$\dot{\boldsymbol{R}}_\alpha = v_\parallel \boldsymbol{b} + c\frac{m_\alpha v_\parallel^2}{q_\alpha B}(\nabla\times\boldsymbol{b})_\perp + c\frac{\boldsymbol{b}\times(q_\alpha\nabla\Phi + \mu\nabla B)}{q_\alpha B}$$

$$\dot{v}_{\parallel,\alpha} = -\frac{1}{m_\alpha}\boldsymbol{b}\cdot(q_\alpha\nabla\Phi + \mu\nabla B) - c\frac{v_\parallel}{q_\alpha B}(\nabla\times\boldsymbol{b})_\perp\cdot(q_\alpha\nabla\Phi + \mu\nabla B)$$

Fluid velocity: $V = V_{gc} + (c/n)\nabla \times M$

$$V_\alpha = \frac{1}{n_\alpha}\int \frac{2\pi}{m_\alpha}\dot{\boldsymbol{R}}_\alpha f B dv_\parallel d\mu - \frac{1}{n_\alpha e}\left[\nabla\times\left(\boldsymbol{b}\int\frac{2\pi}{m_\alpha}\mu f B dv_\parallel d\mu\right)\right]$$

Verification I: COGENT recovers linear local growth rate

Low beta	Moderately steep density	Uniform T_0 and Φ_0	Local mode
$B_0 \propto 1/R$	$L_n/R = 0.5,\ L_n \equiv -n_0^{-1} dn_0/dR$	$T_{i0} = T_{e0} = const,\ \Phi_0 = 0$	$\rho_i/R = 4\times10^{-3}$

Long-wavelength dispersion relation can be derived*

$$k_\perp^2 \frac{m_i c^2}{B^2} = \sum_\alpha \frac{q_\alpha^2}{T_\alpha}\left\{-1 - \pi\left(-\frac{r}{2L_n} + W_\alpha\right) e^{-2W_\alpha}[\mathrm{Erfc}(-W_\alpha)]^2\right\} \qquad W_\alpha = \frac{\omega q_\alpha r B}{2T_\alpha k_z c}$$

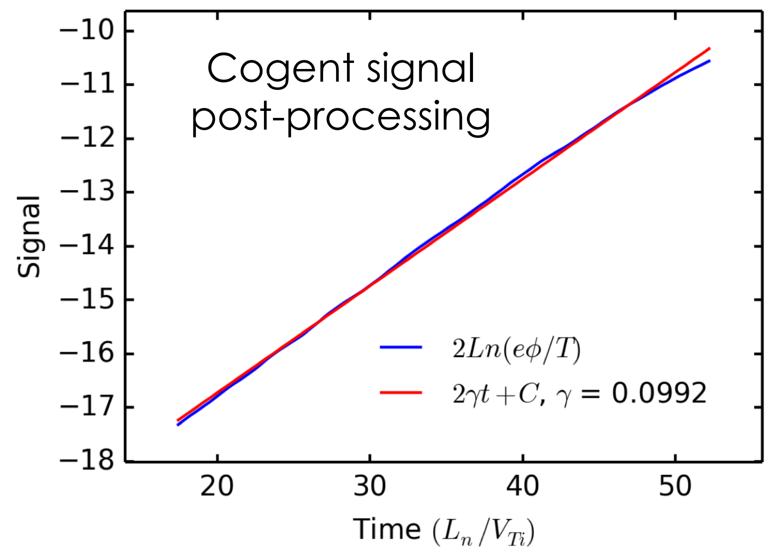
COGENT agrees well with analytics

$$\gamma_{COGENT} = 0.099\, L_n/V_{Ti}$$

$$\gamma_{Analytics} = 0.103\, L_n/V_{Ti}$$

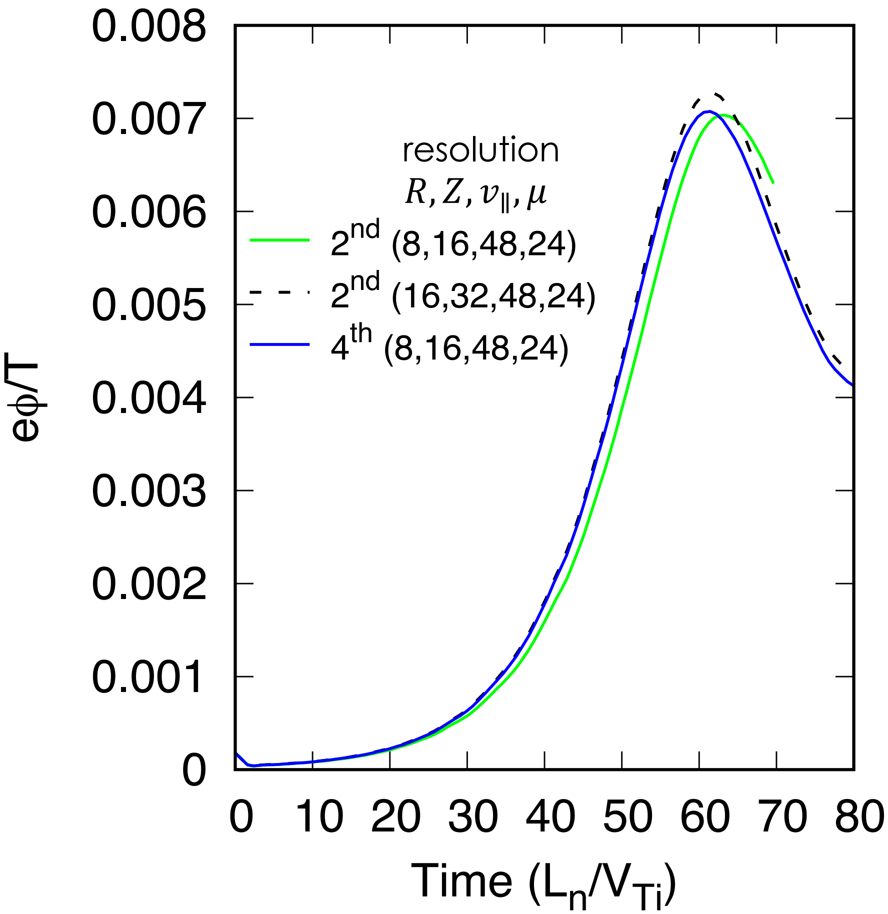
$$k_z\rho_i = \sqrt{2},\ k_r\rho_i = 1/\sqrt{2}$$

$$\rho_i = V_{Ti}\omega_{ci}^{-1} = \omega_{ci}^{-1}\sqrt{2T_i/m_i}$$



*Consistent with the analysis in Ricci et al, PoP 2006

Practical benefits from high-order calculations: faster convergence / effective wall-time usage



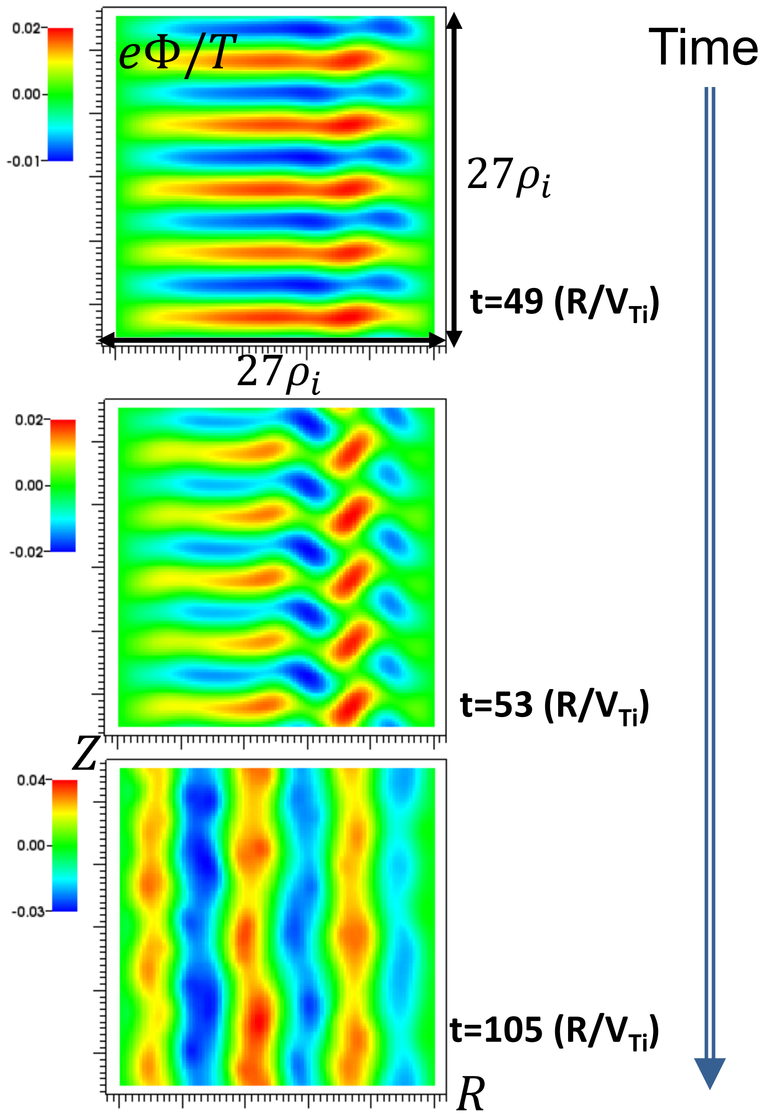
Wall-time to simulate
$60xL_n/V_{ti}$ with 192 cores

4^{th} – order	2^{nd} – order (2X res)
13 mins	43 min
800 steps	1600 steps

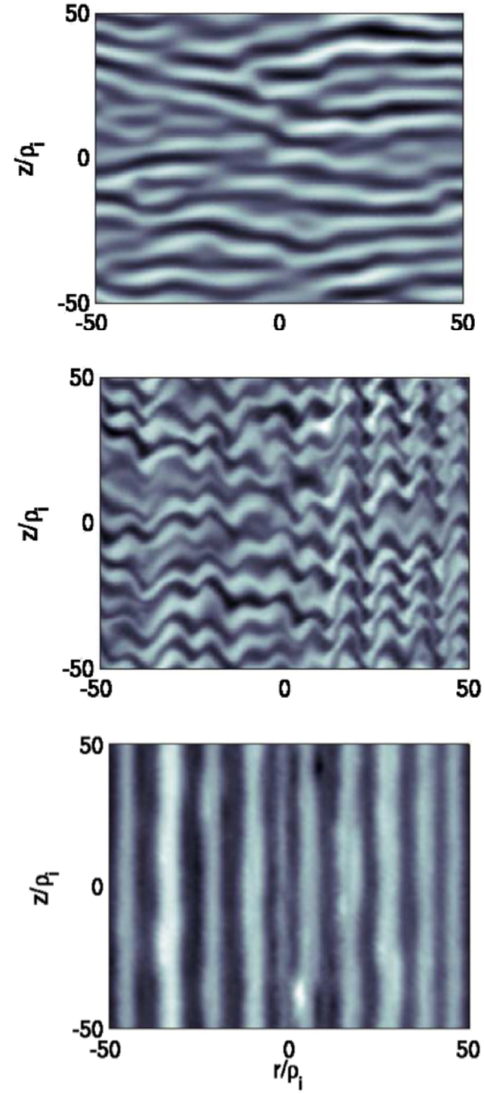
Verification II: nonlinear local simulations recover trends observed in other gyrokinetic simulations*

COGENT
(96x128x48x24)

$L_n/R = 0.77,$
$T_{i0} = T_{e0} = const,$
$\Phi_0 = 0,$
$\rho_i/R = 6\times10^{-3}$



***Ricci et al, PRL 06**



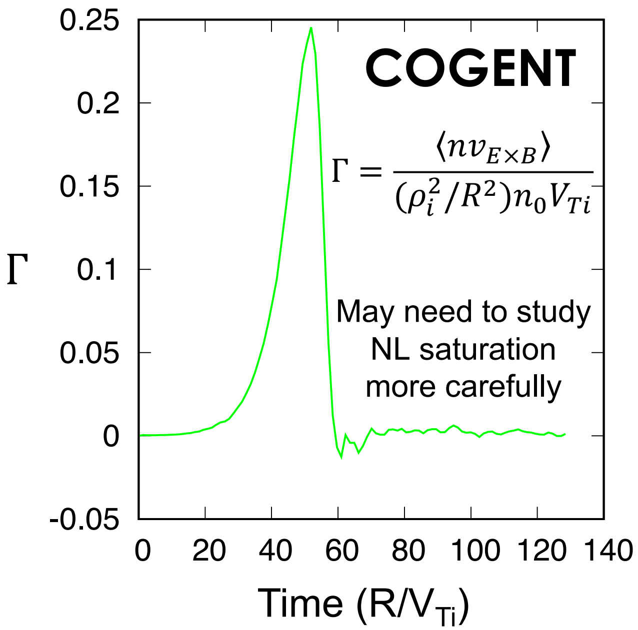
$k_r = 0$ dominates the linear stage of the entropy (primary) mode

Development of $V_r(z)$ triggers KHI with $\rho_i k_r \sim 0.5$

Nonlinear KHI leads to $V_z(r)$ zonal flows that saturate the primary mode

Zonal flows suppress radial transport

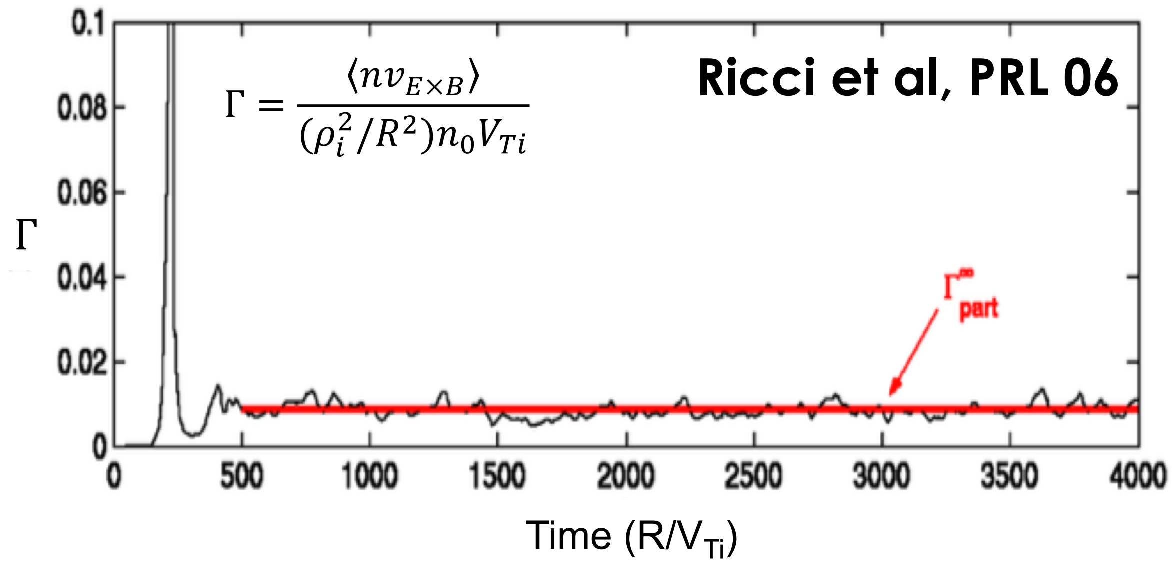
Domain-averaged radial particle flux



$L_n/R = 0.77$, no collisions

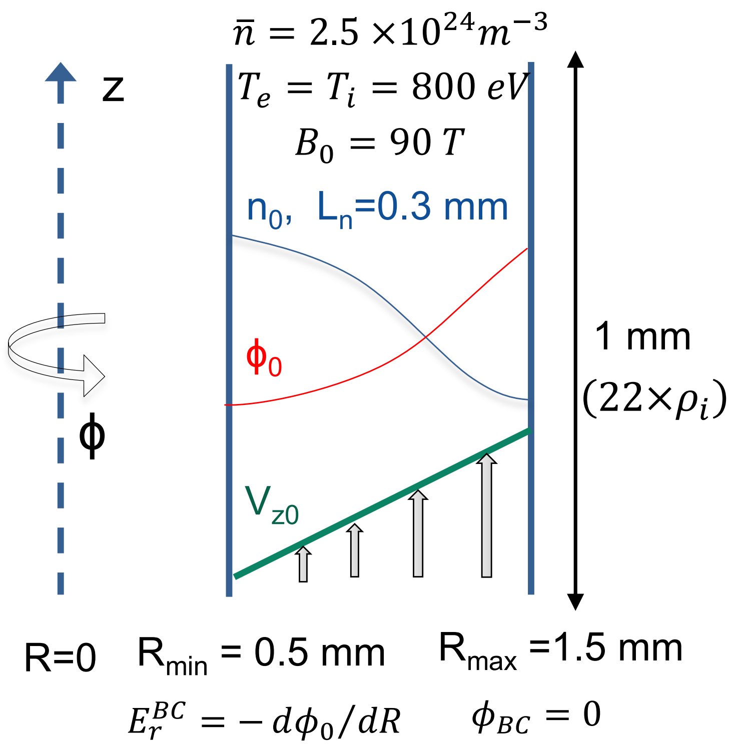
(96x128x48x24) 384 cores X 10 hours

Domain-averaged radial particle flux



$L_n/R = 1,\ \nu_{cls} = 0.01\ V_{Ti}/R$

Entropy mode for FUZE-like parameters: large-amplitude (destructive) perturbations develop

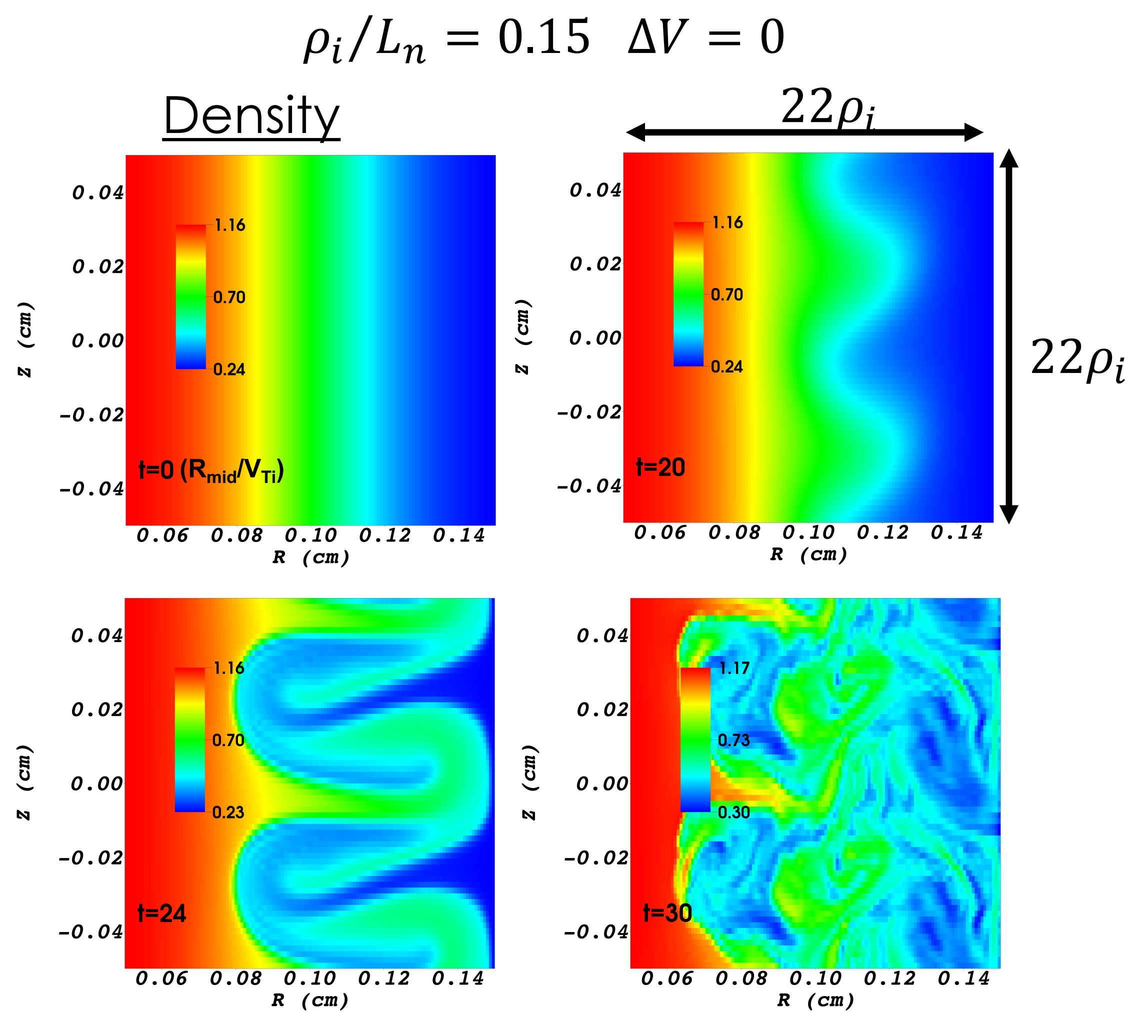


$$n_0(r) = \bar{n}(0.5\times(Tanh[(R - R_{mid})/L_n]) + 0.7)$$

$$V_{z0}(R) = \Delta V\sqrt{T_i/m_i}\,(R - R_{min})/(R_{max} - R_{min})$$

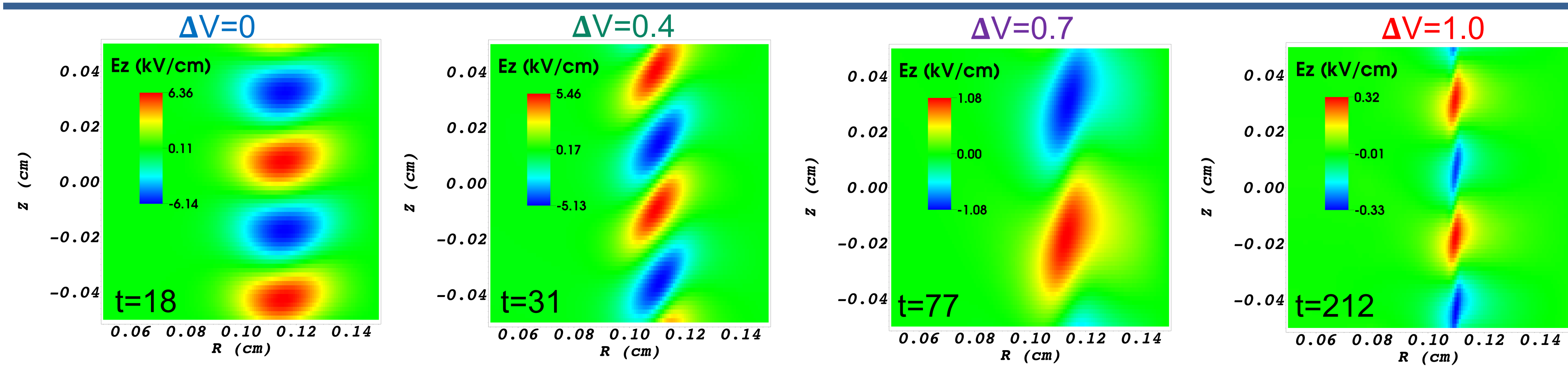
$$T_{e,0}(R) = T_{i,0}(R) = const$$

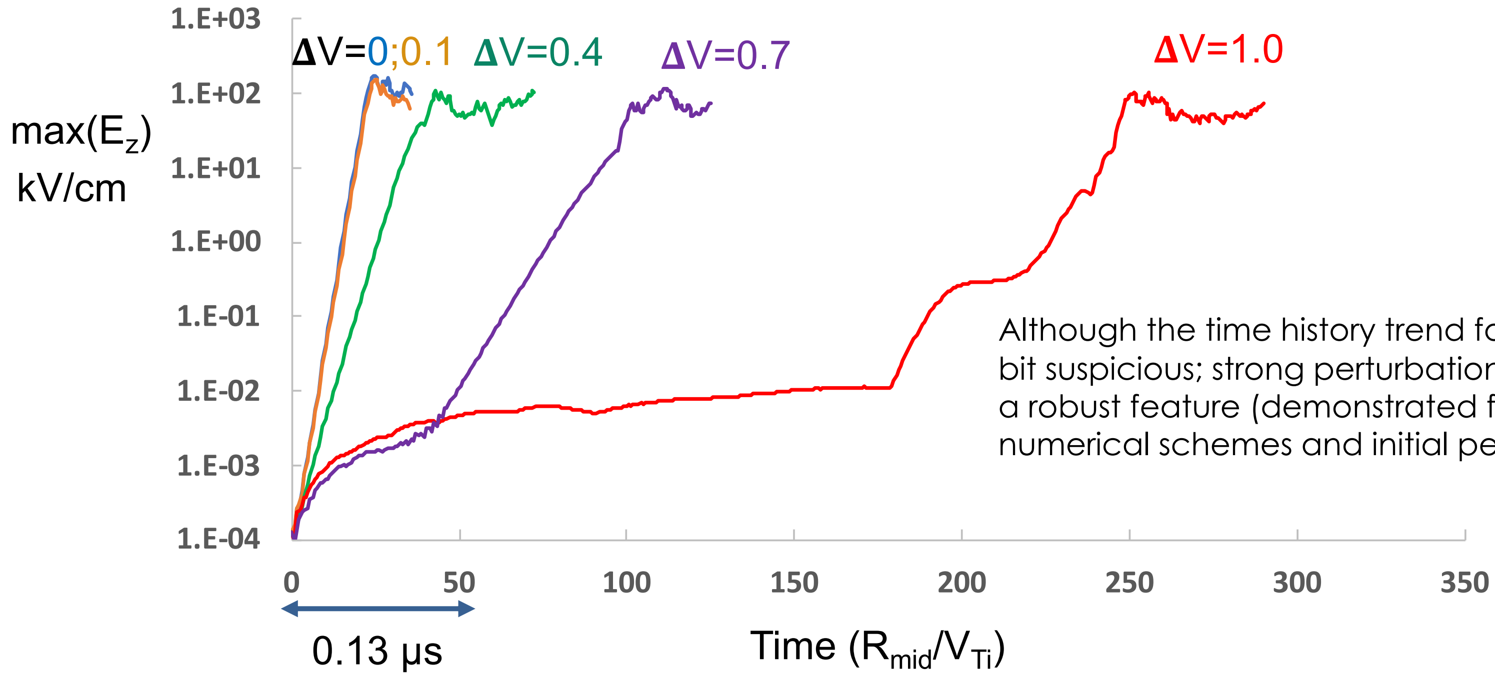
$$B_\phi(R) = const$$



(128x64x48x24) 384 cores X 1 hour

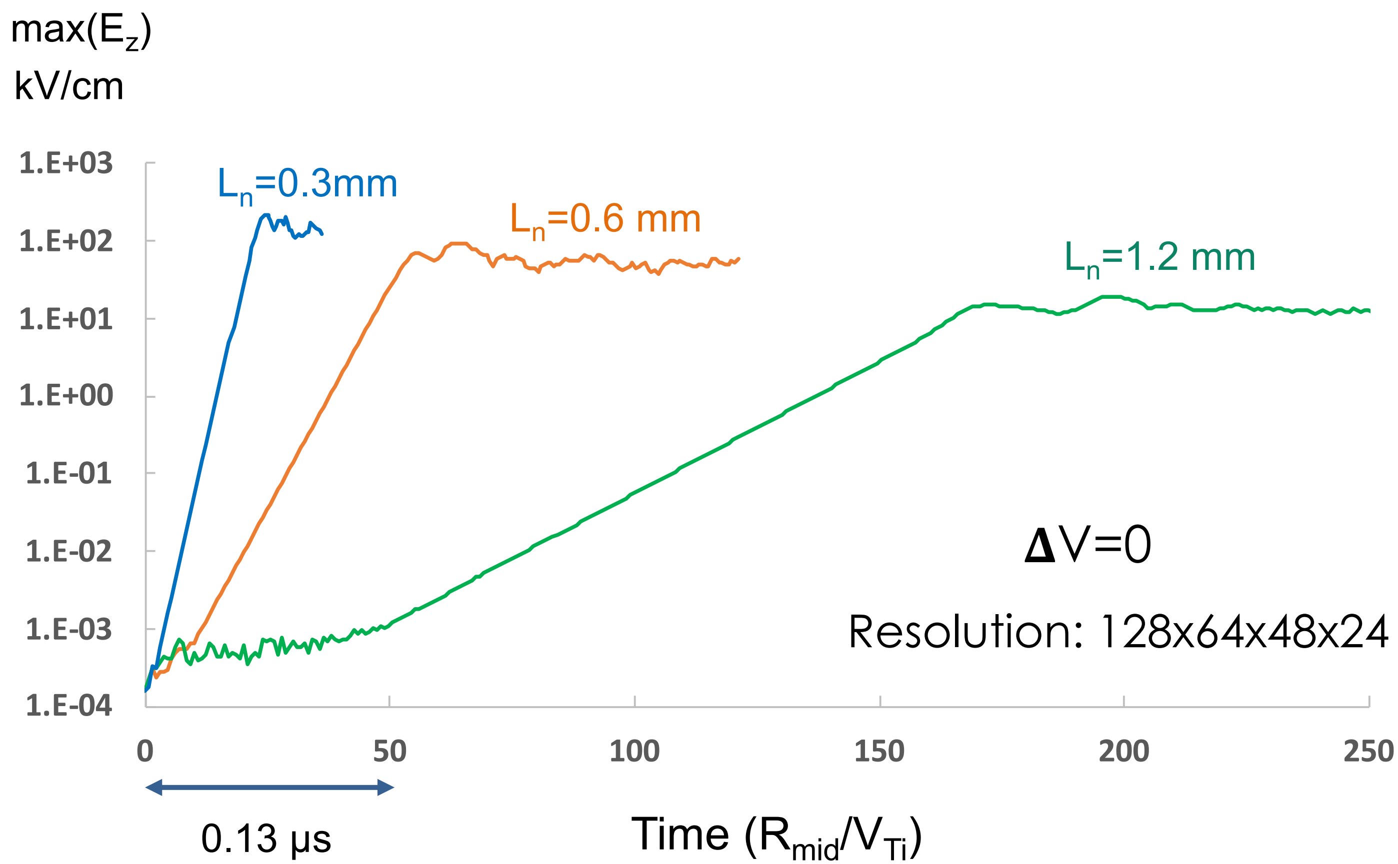
Modest shear has little effect on the instability; even strong shear does not seem to provide stabilization



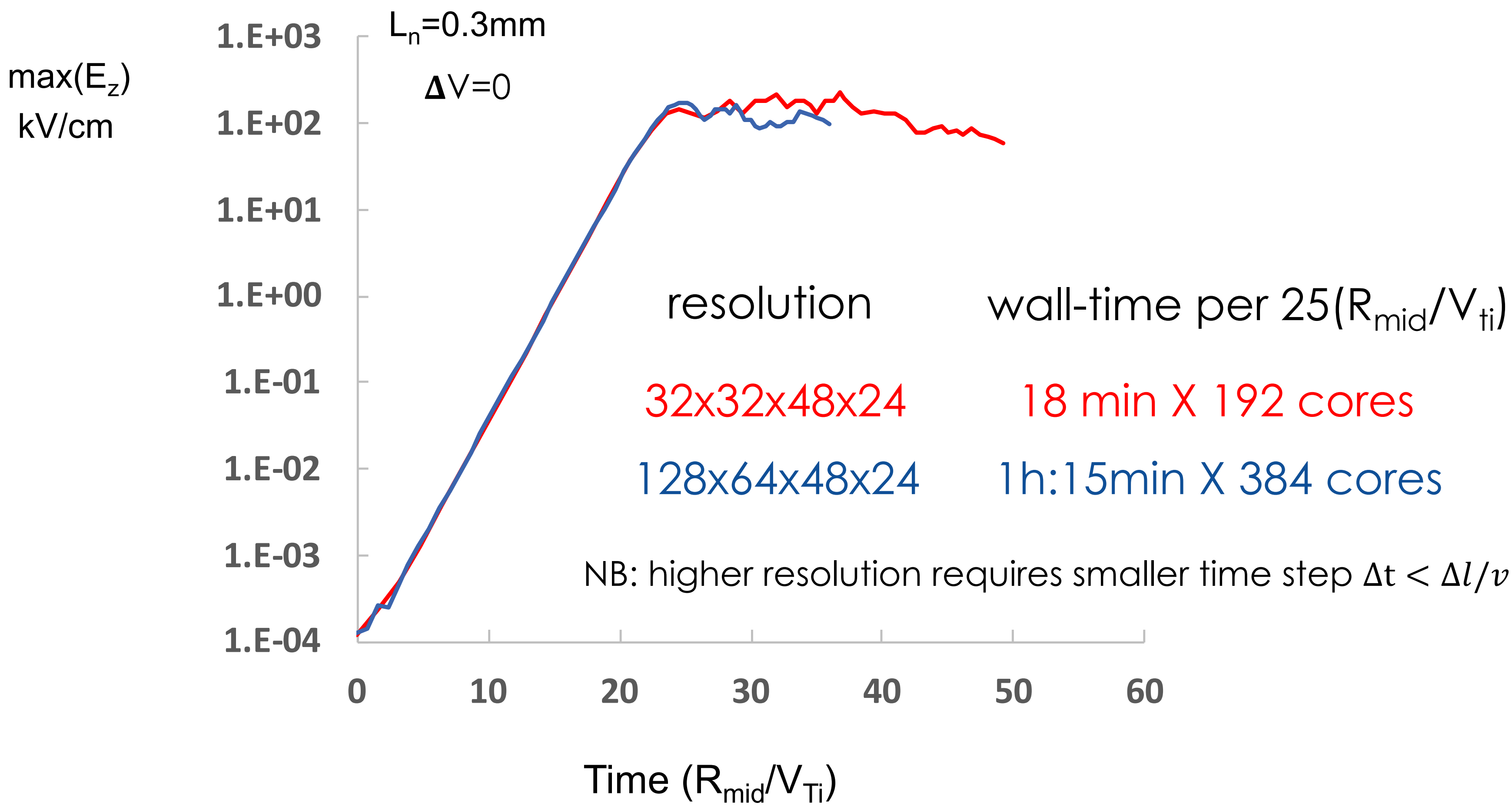


Although the time history trend for **Δ**V=1.0 looks a bit suspicious; strong perturbations appear to be a robust feature (demonstrated for different numerical schemes and initial perturbations)

Effects of micro-instabilities are weaker for shallow-gradient plasmas



Convergence & wall-time (for FUZE-like parameters)



Conclusions/Future work

Conclusions

- Full-F gyrokinetic code COGENT is employed for the analysis of drift-type microturbulence in shear-flow stabilized Z-pinch systems
- The code is successfully verified in a local regime
- Initial results for FUZE-like system parameters demonstrate strong influence of ion-scale drift micro-instabilities

Future work:

- Consider more realistic experimental profiles (ZAP, FUZE, etc)
- Incorporate FLR and EM (finite-β) effects
- Explore gyrokinetic opportunities for other Z-pinch applications
  - Dense plasma focus, fast Z-pinch for ICF
  - Relevant physics problems can involve gyrokinetic analysis of anomalous resistivity generated by lower-hybrid drift microturbulence (occurs on the electron gyro-scale)

Characteristic parameters of FSZP systems

Plasma Conditions	Existing (ZAP)	ARPA-E (FUZE**)	Reactor
Pinch current (kA)	50	300	1500
Total discharge (kA)	150	500	1700
Pinch Radius (mm)	10	0.7	0.05
Ion Density (m^{-3})	1 E+22	2.5 E+24	3 E+27
Temperature	50-100 eV	2500-4000 eV	25-50 keV
Magnetic field (tesla)	1	90	6000
Lawson n-tau (m^{-3} sec)	1 E+17	1 E+19	1 E+21
Derived parameters			
Spatial magnetization*, ρ_i/a	0.1-0.15	0.16-0.2	0.1-0.15
Temporal magnetization*, $V_{Ti}/a\omega_{ci}$	0.1-0.15	0.16-0.2	0.1-0.15
Collisionality, $(1/(\tau_i\gamma) \sim a/\tau_i V_{Ti})$	1.4-0.4	0.01-0.005	0.01-0.003

* Plasma periphery is stronger magnetized than presented values, since $T_{perifery} < T_{core}$ and $L_B > a$

**The FUZE facility will serve as a source of X-rays producing 10 MW of 2 keV radiation with 100 J per 10 μs pulse, as well as a source of neutrons producing 0.16 J/pulse of 2.45 MeV neutrons