Progress in High-Resolution Methods for Continuum Kinetic Models in COGENT[1]

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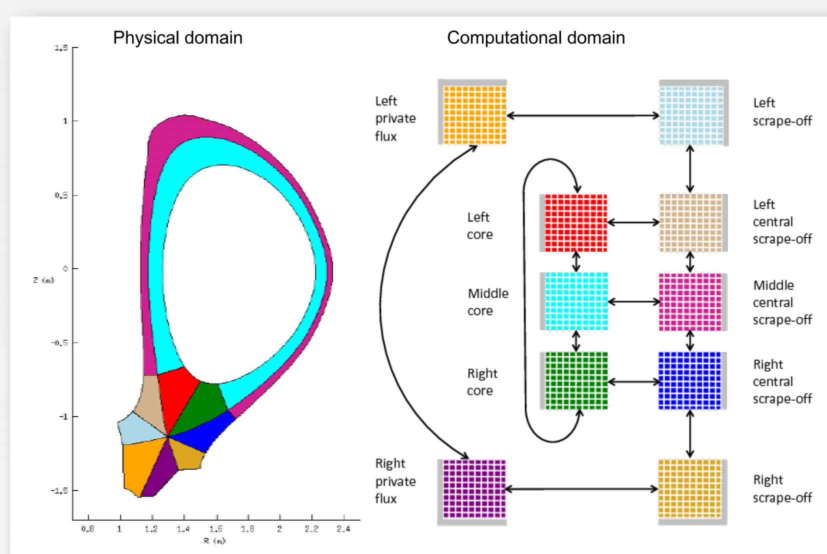
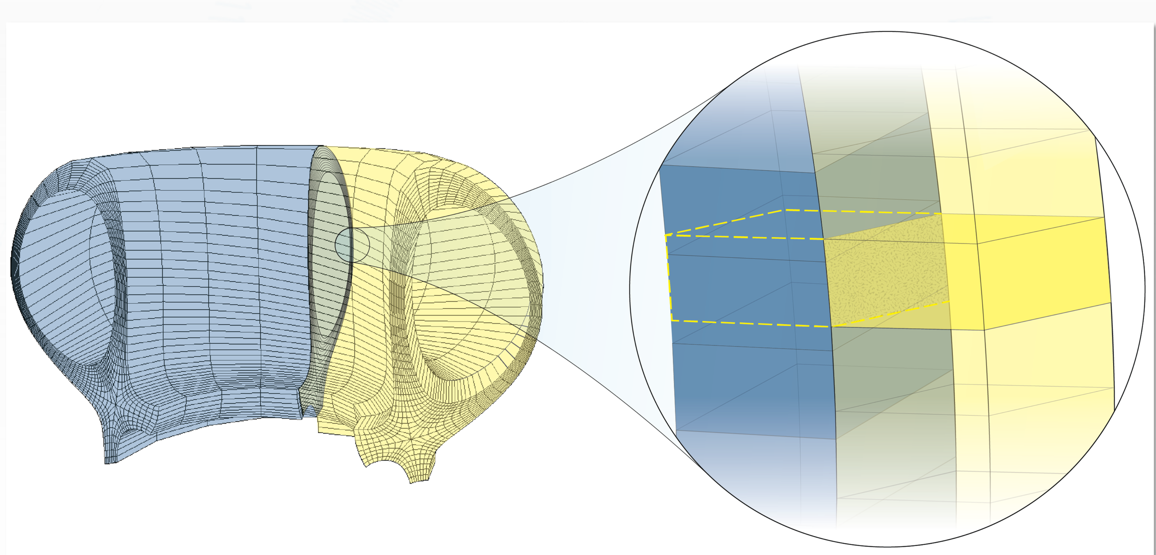
Abstract

In the development of the COGENT code, in collaboration with our Fusion Energy Sciences partners, we have deployed a collection of novel numerical methodologies that have enabled the first ever continuum kinetic simulations in the edge plasma region of tokamak reactors spanning both sides of the magnetic separatrix. We describe here our recent progress in extending the code to 5D, field-aligned geometry, multi-scale time integration, sparse grids, and electromagnetic physics.

Motivation and Approach

Strong anisotropy motivates field-aligned coordinates, but complex geometry necessitates multi-block structure. In 4D, conformal cell faces at block boundaries yielded simplifications… but magnetic shear makes this impossible in 5D.





Novel approach to handle non-conformality in one dimension via least-squares interpolation.

In addition to complex geometry, tokamak physics are multi-scale in time. Treated with IMEX methods and multi-physics preconditioning:

$$\frac{\partial f_i}{\partial t} + \nabla_\parallel \cdot \left(\dot{\mathbf{R}} f_i\right) + \nabla_\perp \cdot \left(\dot{\mathbf{R}} f_i\right) + \partial_{v_\parallel}(\dot{v}_\parallel f_i) = C_i(f_i) + \nabla_\perp \cdot (D_{AN} \nabla_\perp f_i)$$

$$\frac{\partial f_e}{\partial t} + \nabla_\parallel \cdot \left(\dot{\mathbf{R}} f_e\right) + \nabla_\perp \cdot \left(\dot{\mathbf{R}} f_e\right) + \partial_{v_\parallel}(\dot{v}_\parallel f_e) = C_e(f_e) + \nabla_\perp \cdot (D_{AN} \nabla_\perp f_e)$$

Faster ↕ Slower

Time scale	ω
Electrostatic plasma waves	$V_{T_e} k_\parallel / \rho_s k_\perp$
Weakly-collisional parallel electron transport	$V_{T_e} k_\parallel$
Ion acoustic mode and parallel ion transport	$\sqrt{T/m_i} k_\parallel$
Plasma perpendicular drift	$\frac{\rho_i}{L} V_{T_i} k_\perp$
Transport (profile evolution)	$D_{AN} k_\perp^2,\ \nu \rho_i^2 k_\perp^2$

$$\mathcal{I} - \Delta t \sum_k \mathcal{J}_k \approx \prod_k (\mathcal{I} - \Delta t \mathcal{J}_k)$$

Linear operator from JFNK — Precondition each of these terms

Number of terms needing implicitness is problem dependent… unique preconditioner for every combination impractical. Our solution: operator splitting.

Sparse grids: Addressing the "curse of dimensionality" in kinetic simulations by generalizing the combination technique to our high-order, mapped-grid, finite-volume discretization.

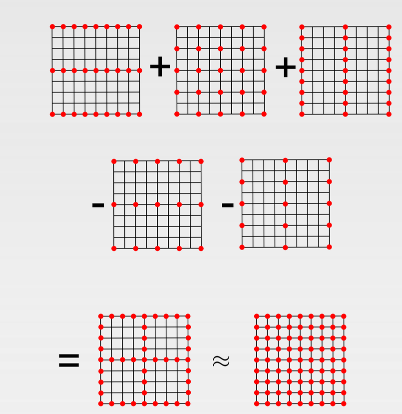
For a p^{th}-order discretization on a mesh with cell size h in d dimensions

$\kappa \propto h^{-d}$ (Complexity) $\quad \epsilon \propto h^p$ (Error) $\Rightarrow \quad \kappa \propto \epsilon^{-d/p}$ (Efficiency)

Required error form:

$$f - \tilde{f} = C_1(h_x) h_x^p + C_2(h_y) h_y^p + C_3(h_x, h_y) h_x^p h_y^p$$

Solution is computed on semi-coarsened grids and combined:

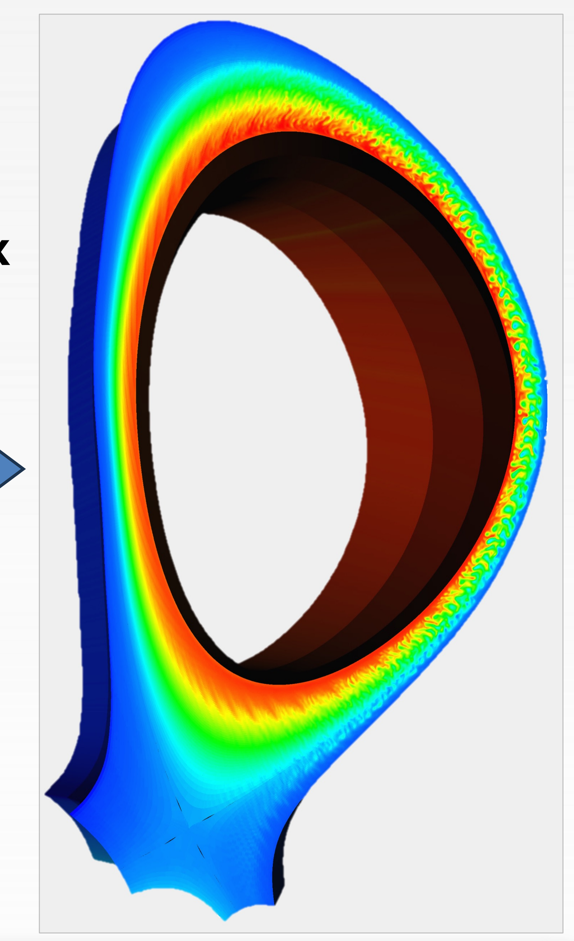


$$\kappa \propto h^{-1} |\log(h)|^{d-1}$$
$$\epsilon \propto h^p |\log(h)|^{d-1}$$
$$\kappa \propto \epsilon^{-1/p} |\log(\epsilon)|^{(d-1)(1+1/p)}$$

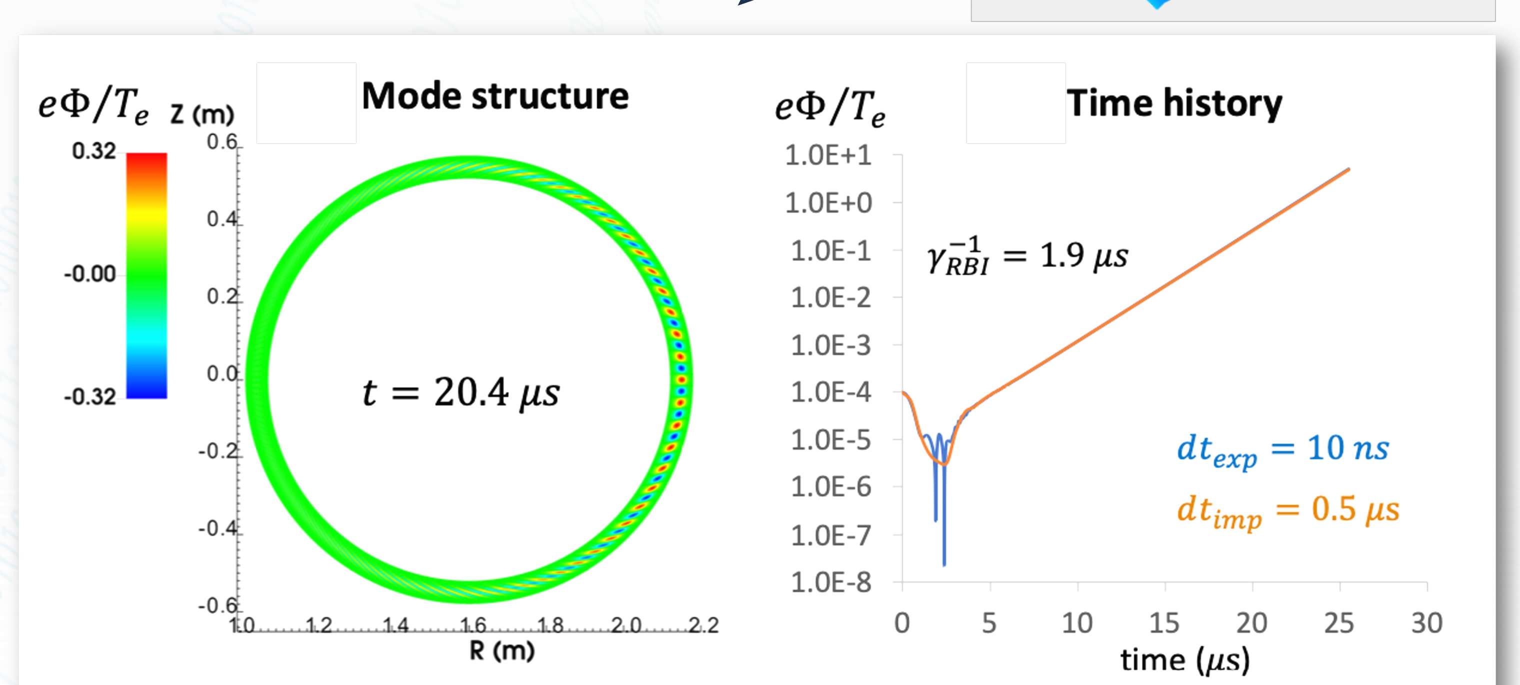
Anisotropy & mapped grids are synergistic!

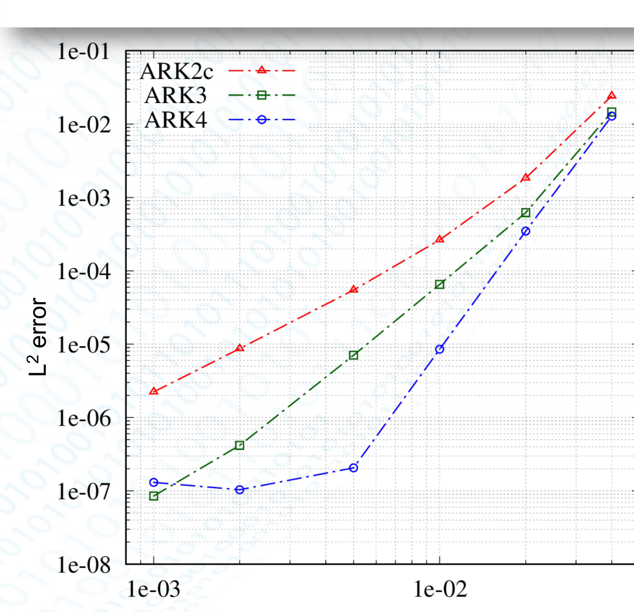
Preliminary Results

World's first continuum, cross-separatrix tokamak turbulence simulations, now in realistic DIII-D and TCV geometries



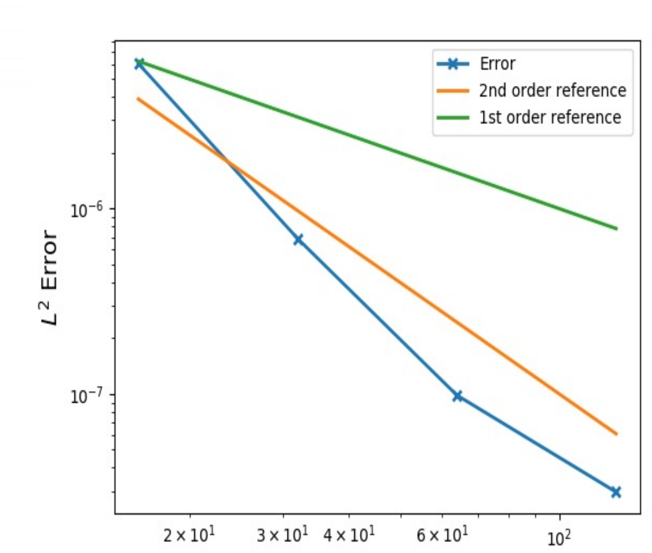
Generalized field solver from electrostatic to electromagnetic. IMEX schemes + novel preconditioner enable 50x increase in time-step for kinetic ballooning instability.

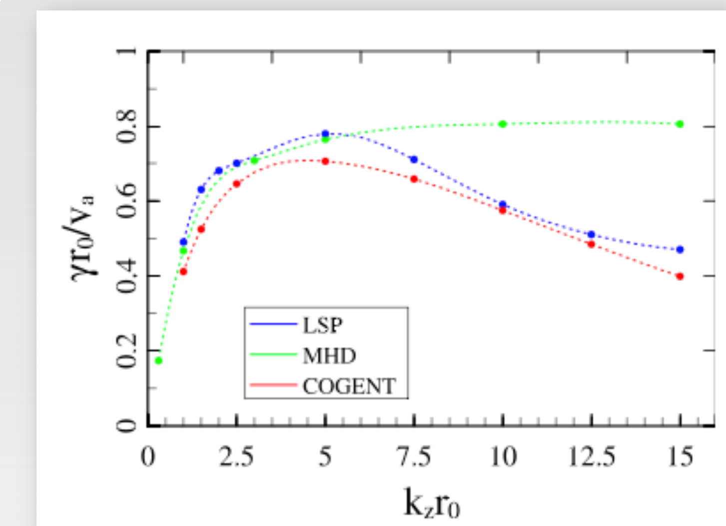




IMEX w/ multiphycis preconditioning convergence in DIII-D H-mode test case

Sparse grids on 4D drift-wave problem. Higher order and improved parallelism in progress



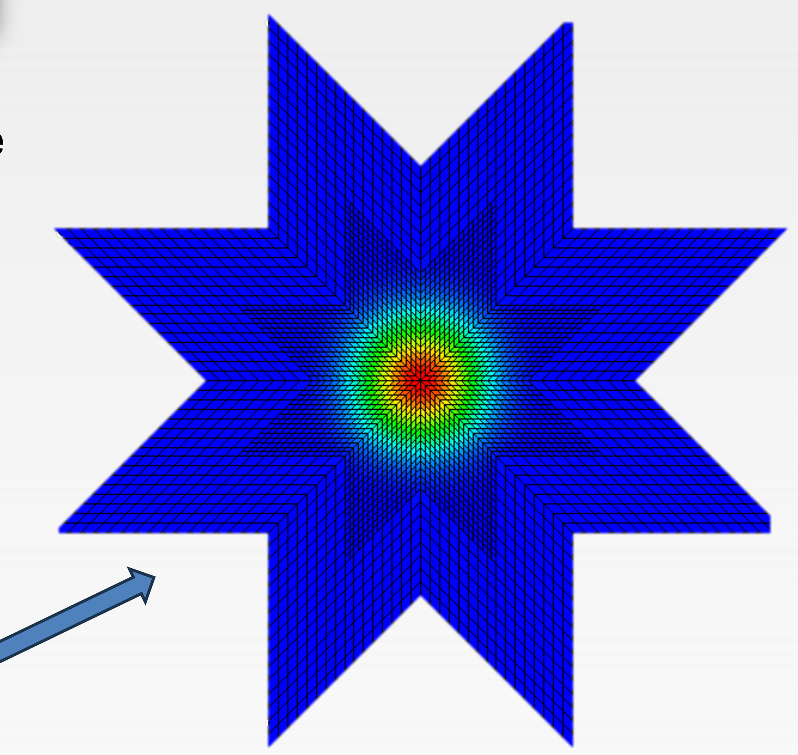


COGENT's flexible design enables quick adaptation to other applications:

- Z-pinch plasmas (see left)
- MHD
- Magnetic mirrors (very new)

Other ongoing activities:

- Rosenbrock-Krylov (ROK) time integrators – potentially mitigate need for custom preconditioners
- Embedded boundary methods at divertor plates
- GPU support via new Chombo performance portability layer
- Mapped-grid mesh refinement near X-point, where fine-scale solution structures develop



Impacts

Implementation of our algorithms in COGENT provides a unique capability for the fusion sciences community through recently completed ASCR/FES SciDAC partnerships:

- Advanced Tokamak Modeling (AToM)
- Plasma Surface Interaction (PSI)

Addtionally, LLNL LDRDs on

- Applying the gyrokinetic formulation of magnetized particle dynamics to Z-pinch plasmas
- SI proposal for data driven reduced order modeling w/ LibROM team, with COGENT providing high-fidelity data (see CHaRMNET MMICC talk by Youngsoo Choi)

Goal: Predictive simulations of current- and next-generation tokamaks

References

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